

Lecture 3
2020/2021

Microwave Devices and Circuits for Radiocommunications

2020/2021

- 2C/1L, MDCR
- Attendance at minimum 7 sessions (course + laboratory)
- Lectures- associate professor Radu Damian
 - Wednesday 15-17, Online, Microsoft Teams
 - E – 50% final grade
 - problems + (2p atten. lect.) + (3 tests) + (bonus activity)
 - 3p=+0.5p
 - all materials/equipments authorized

2020/2021

- Laboratory – associate professor Radu Damian
 - Thursday **10-12**, II.13 / (**Online**)
 - L – 25% final grade
 - ADS, 4 sessions
 - Attendance + personal results
 - P – 25% final grade
 - ADS, 2 sessions (**-1~25.02.2021**)
 - personal homework

Materials

■ <http://rf-opto.etti.tuiasi.ro>

The screenshot shows a web browser displaying the website http://rf-opto.etti.tuiasi.ro/microwave_cd.php?ch_lang=0. The page title is "Microwave Devices and Circuits for Radiocommunications (English)". The main content area includes sections for Course (MDCR 2017-2018), Activities, Evaluation, Grades, Attendance, Lists, and Materials. The right side features the RF-OPTO logo, a globe graphic, and language links (English, Romana). A red circle highlights the "English" link.

Laboratorul de Microunde și Optică

Main Courses Master Staff Research Students Admin

Microwave CD Optical Communications Optoelectronics Internet Antennas Practica Networks Educational software

Microwave Devices and Circuits for Radiocommunications (English)

Course: MDCR (2017-2018)

Course Coordinator: Assoc.P. Dr. Radu-Florin Damian
Code: EDOS412T
Discipline Type: DOS; Alternative, Specialty
Credits: 4
Enrollment Year: 4, Sem. 7

Activities

Evaluation

Type: Examen

A: 50%, (Test/Colloquium)
B: 25%, (Seminary/Laboratory/Project Activity)
D: 25%, (Homework/Specialty papers)

Grades

[Aggregate Results](#)

Attendance

[Course](#)
[Laboratory](#)

Lists

[Bonus-uri acumulate \(final\)](#)
[Studentii care nu pot intra in examen](#)

Materials

Course Slides

[MDCR Lecture_1 \(pdf, 5.43 MB, en, !\[\]\(5bd3139e49b8ec618dddaa46174de8b0_img.jpg\)](#)
[MDCR Lecture_2 \(pdf, 3.67 MB, en, !\[\]\(9aae4ef11f04080694e1bcd3250dc654_img.jpg\)](#)
[MDCR Lecture_3 \(pdf, 4.76 MB, en, !\[\]\(1f875e8ff0db454eb302861a56ff194f_img.jpg\)](#)
[MDCR Lecture_4 \(pdf, 5.58 MB, en, !\[\]\(05604d380e755a92e3161ab249a7c58e_img.jpg\)](#)

 ETI

RF-OPTO



 English |  Romana |

Main Courses Master Staff Research

Grades Student List Exams Photos

Online Exams

In order to participate at online exams you must get ready following

Materials

- RF-OPTO
 - <http://rf-opto.eti.tuiasi.ro>
- **David Pozar, “Microwave Engineering”,**
Wiley; 4th edition , 2011
 - 1 exam problem ← Pozar
- Photos
 - sent by ~~email~~/online exam
 - used at lectures/laboratory

Profile photo

- Profile photo – online “exam”

Examene online: 2020/2021

Disciplina: MDC (Microwave Devices and Circuits (Engleza))

Pas 3

Nr.	Titlu	Start	Stop	Text
1	Profile photos	03/03/2021; 10:00	08/04/2021; 08:00	Online "exam" created f ..
2	Mini Test 1 (lecture 2)	03/03/2021; 15:35	03/03/2021; 15:50	The current test consis ..

Online

- access to **online exams** requires the **password** received by email

English | Romana |

Main Courses Master Staff Research **Student List**

Grades Student List Exams Photos

POPESCU GOPO ION

Fotografia nu există

Date:

Grupa	5700 (2019/2020)
Specializarea	Inginerie electronica si telecomunicatii
Marca	7000000

[Access the site as this student](#) | [Request access to software](#)

Grades

Inca nu a fost notat.

Main Courses Master Staff Research

Grades **Student List** Exams Photos

Login

Use the last name and email stored in the database

Name
POPESCU GOPO

Email/Password

Write the code below

828f26b

Send

Password

■ received by email

Important message from RF-OPTO Inbox x

Radu-Florin Damian
to me, POPESCU ▾

Romanian ▾ English ▾ Translate message

 Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
Universitatea Tehnica "Gh. Asachi" Iasi

In atentia: POPESCU GOPO ION

Parola pentru a accesa examenele pe server-ul rf-opto este
Parola: [REDACTED]

Identificati-vă pe [server](#), cu parola, cat mai rapid, pentru confirmare.

Memorati acest mesaj intr-un loc sigur, pentru utilizare ulterioara

Attention: POPESCU GOPO ION

The password to access the exams on the rf-opto server is
Password: [REDACTED]

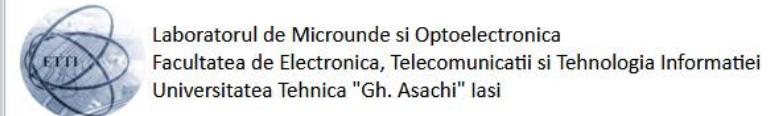
Login to the [server](#), with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

Reply Reply all Forward

Subject: Important message from RF-OPTO Correspondents: POPESCU GOPO ION

From: Me <rdamian@etti.tuiasi.ro> ★
Subject: Important message from RF-OPTO (highlighted)
To: [REDACTED]
Cc: Me <rdamian@etti.tuiasi.ro> ★



In atentia: POPESCU GOPO ION

Parola pentru a accesa examenele pe server-ul rf-opto este
Parola: [REDACTED]

Identificati-vă pe [server](#), cu parola, cat mai rapid, pentru confirmare.

Memorati acest mesaj intr-un loc sigur, pentru utilizare ulterioara

Attention: POPESCU GOPO ION

The password to access the exams on the rf-opto server is
Password: [REDACTED]

Login to the [server](#), with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

Online Exam manual

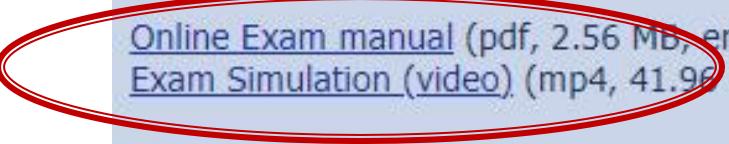
- The online exam app used for:
 - lectures (attendance)
 - laboratory
 - project
 - examinations

Other data

[Online Exam manual \(pdf, 2.56 Mb, en, !\[\]\(15b3c048ece3c5b28384171962f5ff49_img.jpg\)](#)
[Exam Simulation \(video\) \(mp4, 41.96 MB, en, !\[\]\(3b1700cbc304ccaa610d7a6a96d906f3_img.jpg\)](#)

Previous years

2019-2020 2018-2019 2017-2018



Examen online

- always against a **timetable**
 - long period (lecture attendance/laboratory results)
 - short period (tests: 15min, exam: 2h)

Announcement 23:59 (10/05/2020)	Support material 00:05 (11/05/2020)	Exam Topics 00:07 (11/05/2020)	Results 00:10 (11/05/2020)	End 00:20 (15/05/2020)	Confirmation 00:20 (16/05/2020)	Next timeframe in: 05 m 43 s Refresh now
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Announcement

This is a "fake" exam, introduced to familiarize you with the server interface and to perform the necessary actions during an exam: thesis scan, selfie, use email for co...

Server Time

All exams are based on the server's time zone (it may be different from local time). For reference time on the server is now:

10/05/2020 23:59:16

Online results submission

- many numerical values/files

Schema finala	Rezultate - castig	Rezultate - zgromot	Fisier justificare calcul (factor andrei)	Fisier zap (optional)	T1, fisier parmetri S	T2, fisier parmetri S	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Ze1	Zo1	Ze2	Zo2	Ze3	Zo3	Ze4	Zo4	Ze5	Zo5	Ze6
86 - 5428 - 259 ...	86 - 5428 - 260 ...	86 - 5428 - 261 ...	86 - 5428 - 316 ...	-	86 - 5428 - 314 ...	86 - 5428 - 315 ...	148.33	155.88	202.12	164.35	180.91	30.29	185.19	79.9	37	68.89	45.14	61.83	45.05	57.97	46.02	61.85	45.05	68.8
86 - 5622 - 259 ...	86 - 5622 - 260 ...	86 - 5622 - 261 ...	86 - 5622 - 316 ...	86 - 5622 - 262 ...	86 - 5622 - 314 ...	86 - 5622 - 315 ...	26.97	153.5	34.64	35.79	55.56	26.212	10.693	0	0	0	0	0	0	0	0	0	0	0
86 - 5488 - 259 ...	86 - 5488 - 260 ...	86 - 5488 - 261 ...	86 - 5488 - 316 ...	86 - 5488 - 262 ...	86 - 5488 - 314 ...	86 - 5488 - 315 ...	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
86 - 5391 - 259 ...	86 - 5391 - 260 ...	86 - 5391 - 261 ...	86 - 5391 - 316 ...	-	-	-	50	50	50	50	50	50	50	70.14	40.39	61.85	44.59	55.7	45.2	54.89	45.38	58.65	45.8	70.0
86 - 5664 - 259 ...	86 - 5664 - 260 ...	86 - 5664 - 261 ...	86 - 5664 - 316 ...	-	86 - 5664 - 314 ...	86 - 5664 - 315 ...	168.02	150.5	178.28	133.75	92.12	121.67	144.48	94.36	36.19	70.77	42.56	65.69	42.05	55.17	42.29	65.59	42.05	70.7
86 - 5665 - 259 ...	86 - 5665 - 260 ...	86 - 5665 - 261 ...	86 - 5665 - 316 ...	-	86 - 5665 - 314 ...	86 - 5665 - 315 ...	162.2	80.8	209.2	140.85	135.1	183.7	167.6	94.58	36.15	78.16	39.77	65.57	45.05	65.57	45.05	78.16	39.77	94.5
86 - 5433 - 259 ...	86 - 5433 - 260 ...	86 - 5433 - 261 ...	86 - 5433 - 316 ...	-	86 - 5433 - 314 ...	86 - 5433 - 315 ...	165.138	106.228	226.157	130.134	72.71	180.177	164.616	101.36	36.11	77.22	42.49	68.02	45.62	60	45.42	68.02	45.62	77.2
86 - 5608 - 259 ...	86 - 5608 - 260 ...	86 - 5608 - 261 ...	86 - 5608 - 316 ...	-	86 - 5608 - 314 ...	86 - 5608 - 315 ...	150.84	152.5	30.94	32.37	54.36	19.837	29.85	64.14	40.145	54.32	46.32	53.8	46.7	53.8	46.7	54.32	46.32	54.9
86 - 5555 - 259 ...	86 - 5555 - 260 ...	86 - 5555 - 261 ...	86 - 5555 - 316 ...	-	86 - 5555 - 314 ...	86 - 5555 - 315 ...	168.001	150.288	178.399	133.115	92.491	121.257	144.126	97.05	36.16	71.13	43.09	65.45	42.12	55.66	42.18	65.45	42.12	71.1

Online results submission

- many numerical values

i	z1	z2	z3	z4	z5	z6	z7
	148.33	155.88	202.12	164.35	180.91	30.29	185.19
	25.97	153.5	34.64	35.79	55.56	26.212	10,693
	0	0	0	0	0	0	0
	50	50	50	50	50	50	50



Online results submission

Grade = Quality of the work +
+ Quality of the submission

Course Objectives 4



“Engineering”
Sinapses



Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers?~~

Textbooks

- <http://rf-opto.eti.tuiasi.ro>
- Irinel Casian-Botez: "Microunde vol. 1: Proiectarea de circuit", Ed. TEHNOPRES, 2008
- **David Pozar**, Microwave Engineering, Wiley; 4th edition , 2011, ISBN : 978-1-118-29813-8 (E), ISBN : 978-0-470-63155-3 (P)

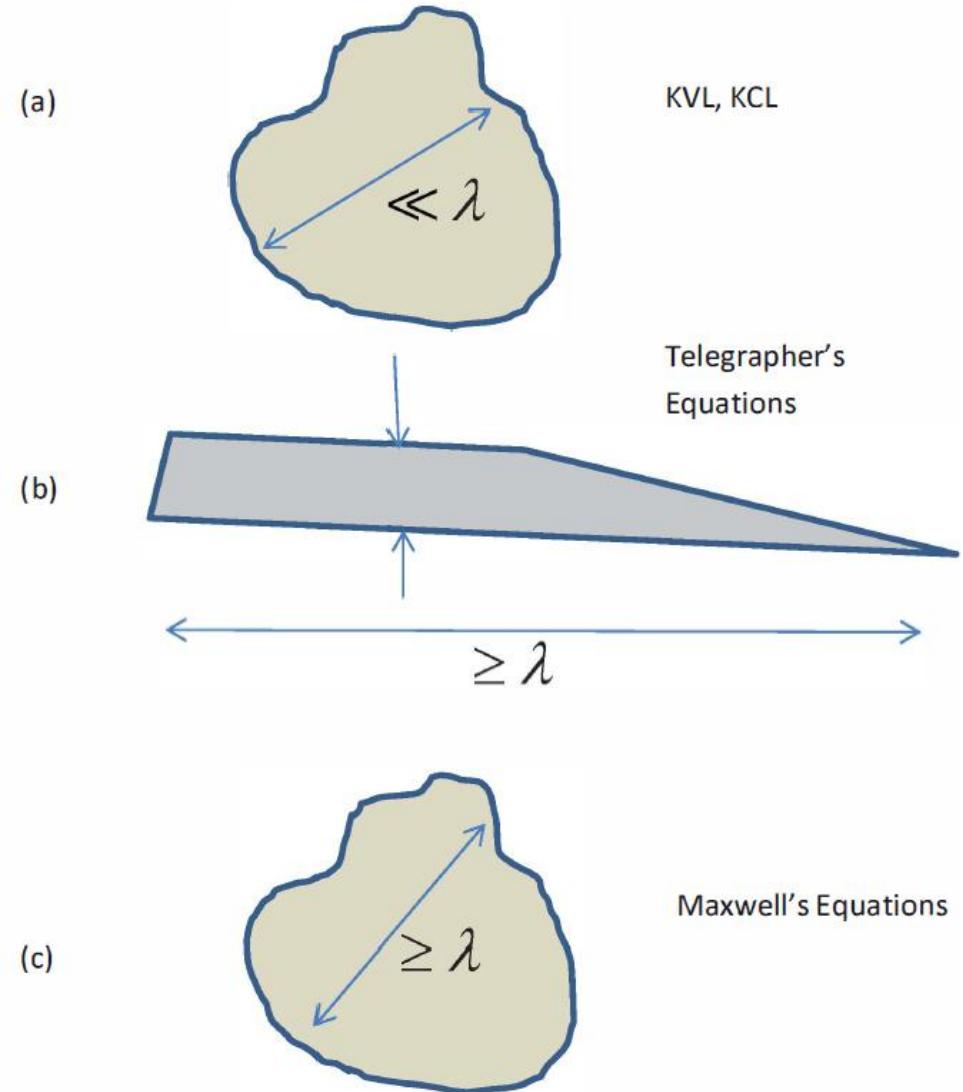
Introduction

Electrical Length

- Behavior (and description) of any circuit depends on his electrical length at the particular frequency of interest

- $E \approx 0 \rightarrow$ Kirchhoff
- $E > 0 \rightarrow$ wave propagation

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left(\frac{l}{\lambda} \right)$$



Maxwell's Equations

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = \frac{\partial D}{\partial t} + J$$

$$\nabla \cdot D = \rho$$

$$\nabla \cdot B = 0$$

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$

■ Constitutive equations

$$D = \epsilon \cdot E$$

$$B = \mu \cdot H$$

$$J = \sigma \cdot E$$

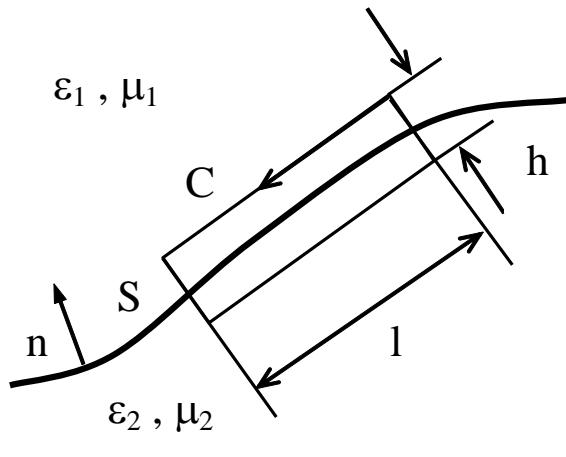
- Vacuum

$$\mu_0 = 4\pi \times 10^{-7} \text{ } H/m$$

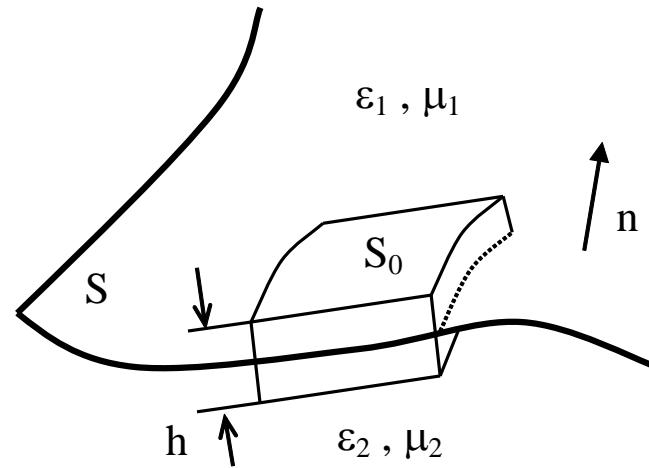
$$\epsilon_0 = 8,854 \times 10^{-12} \text{ } F/m$$

$$c_0 = \frac{1}{\sqrt{\epsilon_0 \cdot \mu_0}} = 2,99790 \cdot 10^8 \text{ } m/s$$

Interface conditions on the interface between two different media



a)



b)

$$n \times (E_1 - E_2) = 0$$

$$n \cdot (D_1 - D_2) = \rho_s$$

$$n \times (H_1 - H_2) = J_s$$

$$n \cdot (B_1 - B_2) = 0$$

- If one of the media is a perfect conductor (metal) all fields are annulled inside

Electromagnetic fields with harmonic time dependence

$$X = X_0 e^{j\omega t} \quad \frac{\partial X}{\partial t} = j\cdot\omega\cdot X$$

$$g(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt \quad f(t) = \int_{-\infty}^{\infty} g(\omega) \cdot e^{j\omega t} d\omega$$

- Maxwell's Equations more simple

$$\nabla^2 E + \omega^2 \epsilon \mu E = j\omega \mu J + \frac{1}{\epsilon} \nabla \rho$$

$$\nabla^2 H + \omega^2 \epsilon \mu H = -\nabla \times J$$

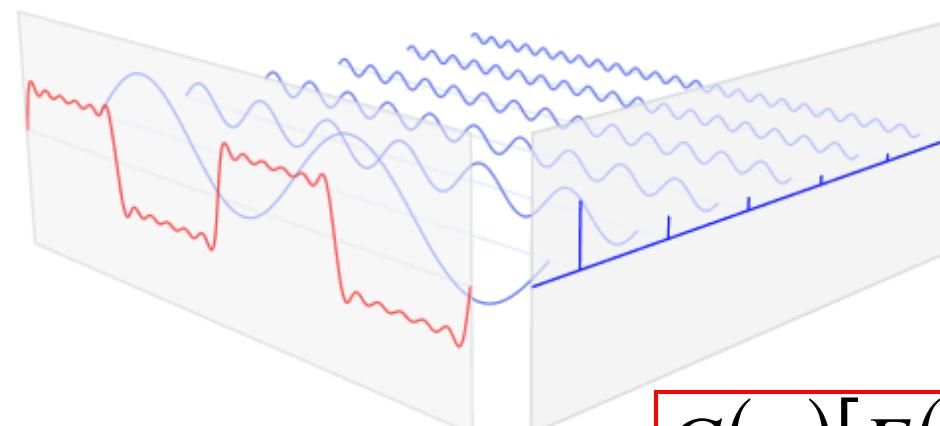
$$\nabla \cdot E = \frac{\rho}{\epsilon}$$

$$\nabla \cdot H = 0$$

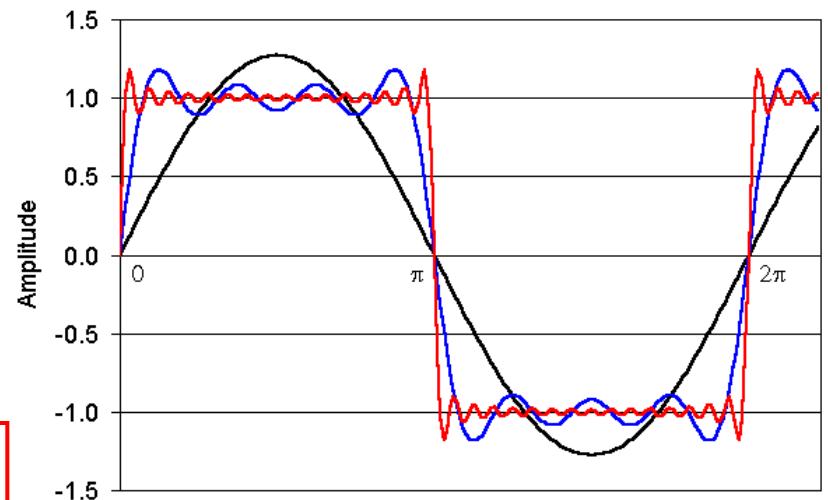
Mathematical models

- particular cases where analytical solution exists
 - harmonic signals, Fourier Transform, frequency spectrum

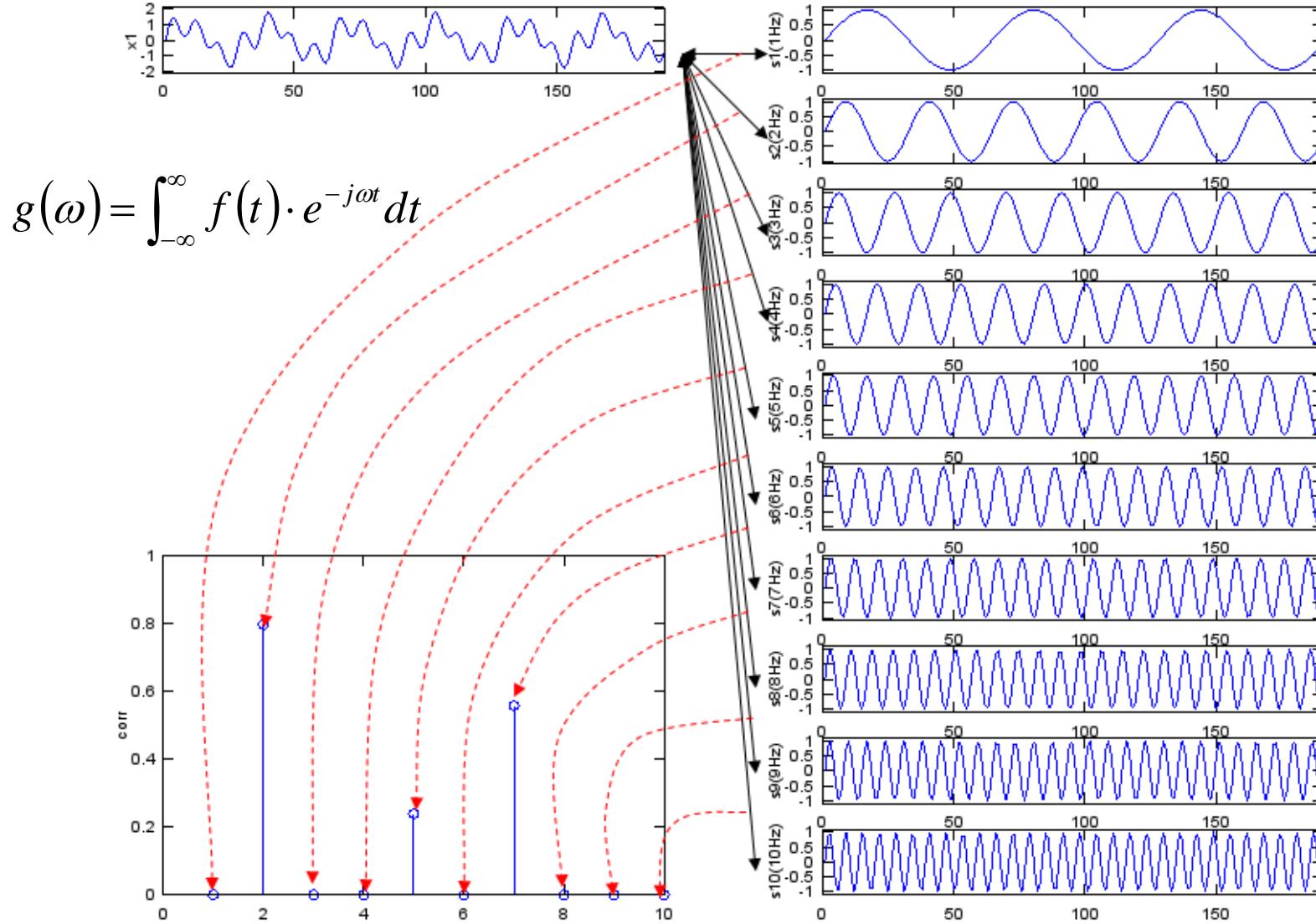
$$X = X_0 e^{j \cdot \omega \cdot t} \quad \frac{\partial X}{\partial t} = j \cdot \omega \cdot X \quad g(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt \quad f(t) = \int_{-\infty}^{\infty} g(\omega) \cdot e^{j\omega t} d\omega$$



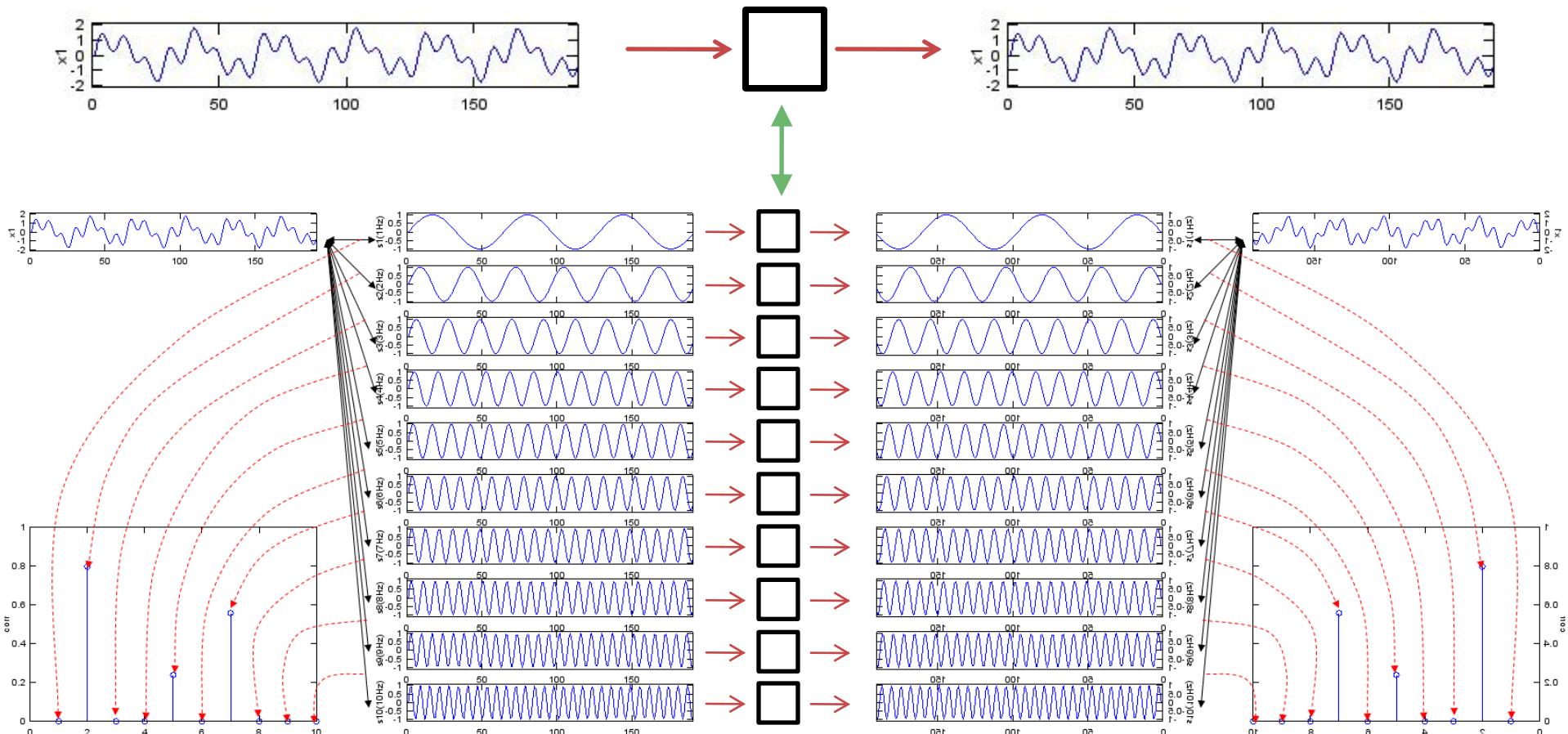
$$G(\omega)[F(\omega)]$$



Mathematical models



Mathematical models



$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt$$

$$G(\omega)[F(\omega)]$$

$$g(t) = \int_{-\infty}^{\infty} G(\omega) \cdot e^{j\omega t} d\omega$$

Wave equations

- Helmholtz equations or Wave equations

Medium void of free electric charges

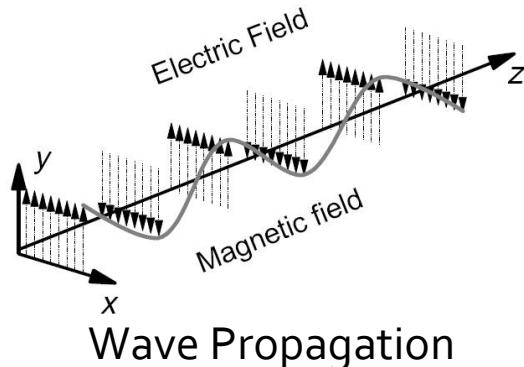
$$\nabla^2 E - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0$$

$$\gamma^2 = -\omega^2 \epsilon \mu + j \omega \mu \sigma$$

γ – propagation constant (known also as phase constant or wave number)

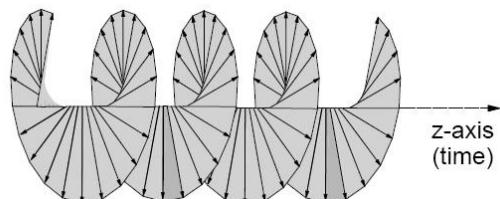
Solutions of the wave equations



Electric field only in Oy direction, ← through judicious choice
wave traveling after Oz direction ← of the coordinate system

$$E_y = E_+ e^{-\gamma \cdot z} + E_- e^{\gamma \cdot z}$$

$$\gamma = \sqrt{-\omega^2 \epsilon \mu + j \omega \mu \sigma} = \alpha + j \cdot \beta$$



Circular Polarization

If we have only the positive direction wave $E_+ \Rightarrow A$

$$E_y = A e^{-(\alpha + j \cdot \beta) \cdot z}$$

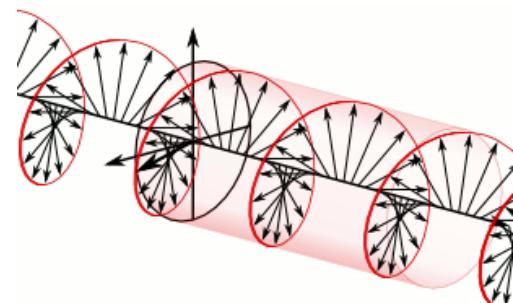
Harmonic Field

$$E_y = A \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)}$$

Amplitude

Attenuation

Wave Propagation
(simultaneous space and
time variation)



Attenuation

$$E_y(z_1) = Ct \cdot e^{-\alpha \cdot z_1} \cdot e^{j(\omega \cdot t - \beta \cdot z_1)}$$

$$E_y(z_2) = Ct \cdot e^{-\alpha \cdot z_2} \cdot e^{j(\omega \cdot t - \beta \cdot z_2)}$$

$$W, P \sim \int E^2$$

$$A = \frac{P_2}{P_1} = \frac{Ct^2 \cdot e^{-2\alpha \cdot z_2}}{Ct^2 \cdot e^{-2\alpha \cdot z_1}} = e^{-2\alpha \cdot (z_2 - z_1)}$$

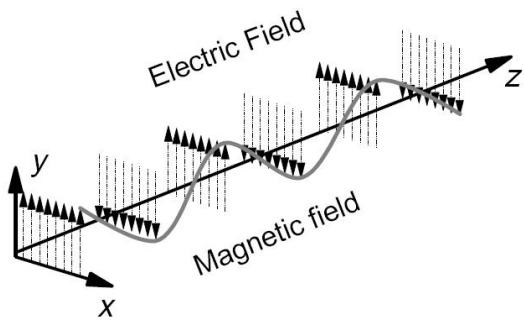
$$A[dB] = 10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} [e^{-2\alpha \cdot (z_2 - z_1)}]$$

$$A[dB] = -20 \cdot \alpha \cdot (z_2 - z_1) \log_{10} e = -8.686 \cdot \alpha \cdot (z_2 - z_1)$$

$$A / L[dB / km] = -8.686 \cdot \alpha < 0$$

- ▶ Attenuation usually expressed in **dB/km**
 - ▶ most of the time a positive value is used
 - ▶ “-” sign = **implied** by the word used

Plane wave parameters



$$\nabla \times E = -j\omega\mu \cdot H$$

$$H_x = \frac{j\gamma \cdot E_y}{\omega\mu}$$

Lossless Medium, $\sigma = 0$

$$\gamma = j\omega \cdot \sqrt{\epsilon\mu}$$

$$\eta = \frac{E_y}{H_x} = \sqrt{\frac{\mu}{\epsilon}} \quad \text{intrinsic impedance of the medium}$$

$$E_y = A \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)}$$

constant phase points: $(\omega \cdot t - \beta \cdot z) = \text{const}$

Phase velocity

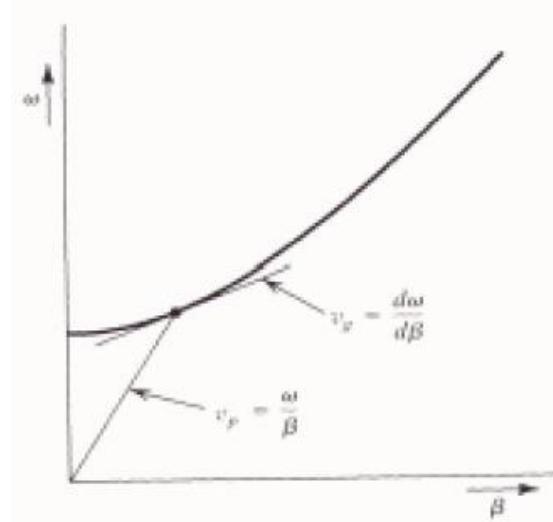
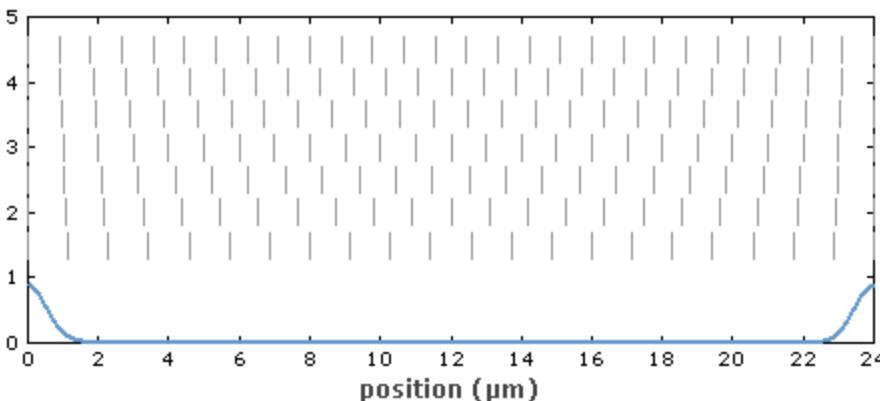
$$v_p = \frac{dz}{dt} = \frac{\omega}{\beta} = \frac{1}{\sqrt{\epsilon\mu}}$$

Group velocity

$$v_g = \frac{dz}{dt} = \frac{d\omega}{d\beta} \quad \text{in dispersive media where } \beta = \beta(\omega)$$

Group and phase velocities

- Phase velocity – **virtual** speed at which a constant phase point travels (in certain conditions might be greater than the speed of light)
- Group velocity – speed at which the signal (energy, information) propagates (always less or equal to the speed of light in that medium)



Plane wave parameters

■ In vid

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\Omega \quad v = v_g = c_0 \quad c_0 = \frac{1}{\sqrt{\epsilon_0 \cdot \mu_0}} = 2,99790 \cdot 10^8 \text{ m/s}$$

$$\lambda_0 = \frac{2\pi}{\beta} = \frac{c_0}{f}$$

Space periodicity

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

Time periodicity

■ In mediu nedispersiv ϵ_r

$$c = \frac{1}{\sqrt{\epsilon \cdot \mu_0}} = \frac{1}{\sqrt{\epsilon_0 \epsilon_r \cdot \mu_0}} = \frac{c_0}{\sqrt{\epsilon_r}}$$

$$n = \sqrt{\epsilon_r} \quad \text{refractive index of a medium}$$

$$c = \frac{c_0}{n}$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f}$$

$$\lambda = \frac{c_0}{\sqrt{\epsilon_r \cdot f}} = \frac{\lambda_0}{\sqrt{\epsilon_r}}$$



Solutions of the wave equations

$$E_y = E^+ e^{-\gamma \cdot z} + E^- e^{\gamma \cdot z}$$

Electric field only in Oy direction, ← through judicious choice
wave traveling after Oz direction ← of the coordinate system

$$\gamma = \sqrt{-\omega^2 \epsilon \mu + j \omega \mu \sigma} = \alpha + j \cdot \beta$$

■ wave

- incident
- reflected

$$E_y = E^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)}$$

$$(\omega \cdot t - \beta \cdot z) = \text{const}$$

■ wave

- direct
- inverse

$$E_y = E^- \cdot e^{\alpha \cdot z} \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

$$(\omega \cdot t + \beta \cdot z) = \text{const}$$

points of
constant
phase

Solutions of the wave equations

■ wave

- incident
- reflected

$$E_y = E^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t - \beta \cdot z)} + E^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t + \beta \cdot z)}$$

$$H_z = H^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t - \beta \cdot z)} + H^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t + \beta \cdot z)}$$

■ wave

- direct
- inverse

$$V(z) = V^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t - \beta \cdot z)} + V^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t + \beta \cdot z)}$$

$$I(z) = I^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t - \beta \cdot z)} + I^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t + \beta \cdot z)}$$

$$V(z) = V^+ \cdot e^{j(\omega t - \beta \cdot z)} + V^- \cdot e^{j(\omega t + \beta \cdot z)}$$

Modes in delimited media

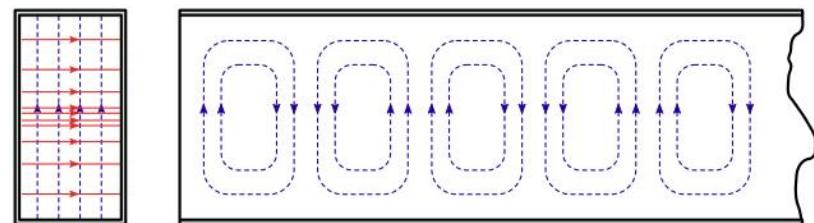
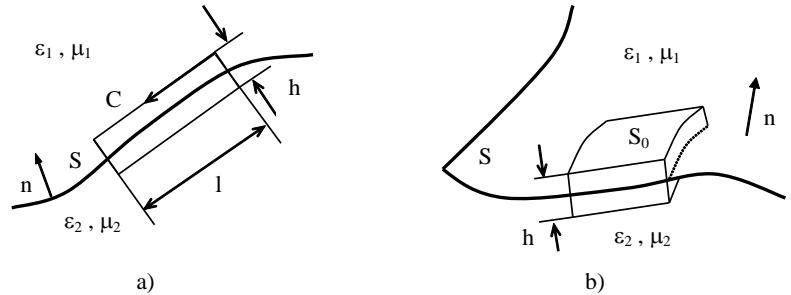
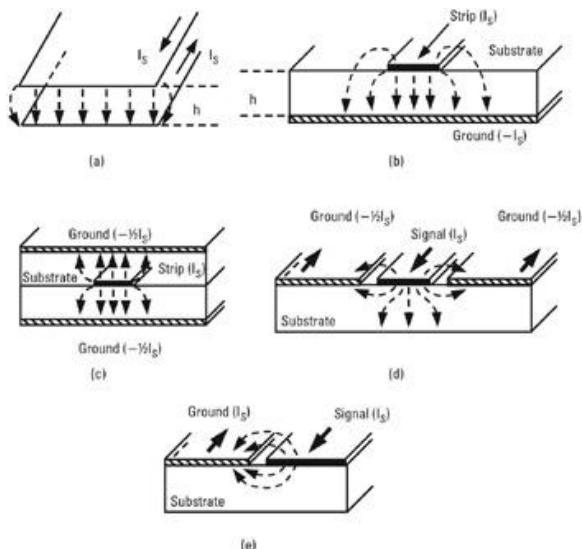
- Electromagnetic fields with harmonic time dependence
 - Maxwell's Equations simplified

$$X = X_0 e^{j\omega t} \quad \frac{\partial X}{\partial t} = j \cdot \omega \cdot X \quad g(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt \quad f(t) = \int_{-\infty}^{\infty} g(\omega) \cdot e^{j\omega t} d\omega$$

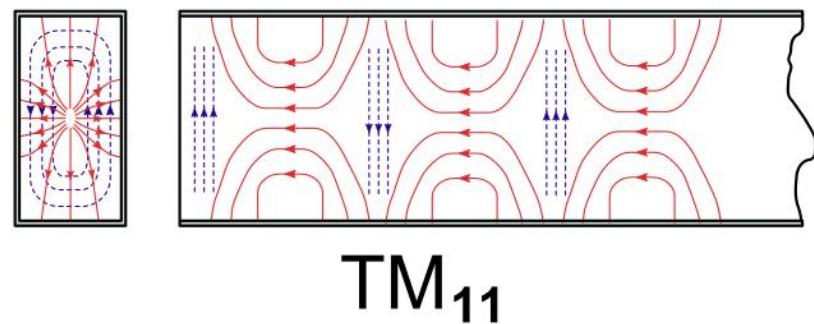
- In delimited media the solutions of Maxwell's Equations must also verify boundary conditions
 - solutions must respect some supplemental conditions

Modes in delimited media

- Electric field must always be normal on an electric wall or annulled
- Magnetic field must always be tangent to an electric wall or annulled

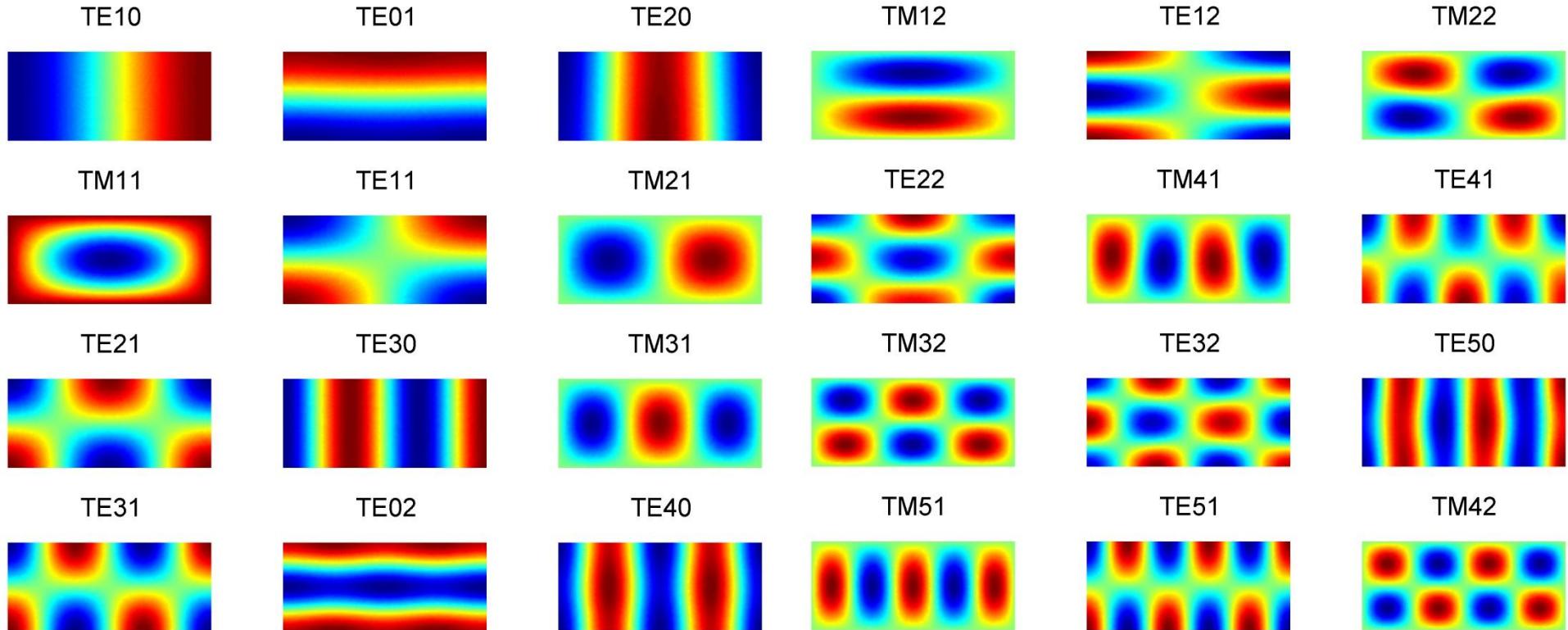


TE₁₀



TM₁₁

Moduri in mediis delimitate



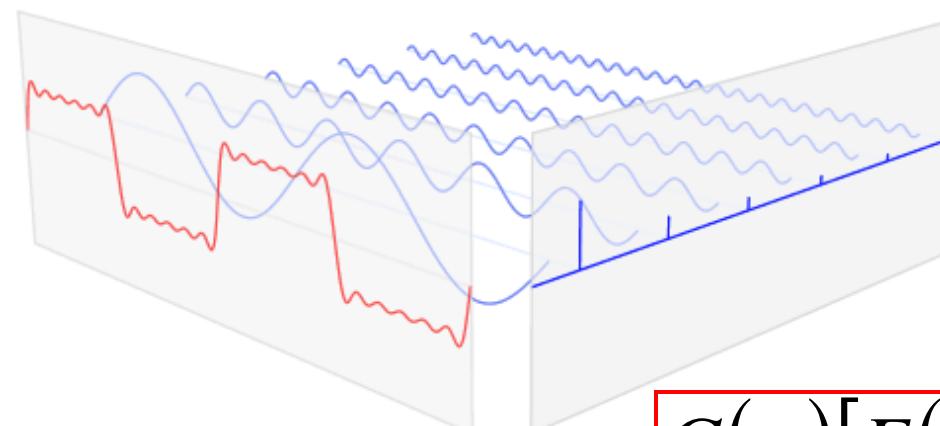
- Similar with Fourier Transform
$$g(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt \quad f(t) = \int_{-\infty}^{\infty} g(\omega) \cdot e^{j\omega t} d\omega$$

$$E^+, E^- = \sum_1^{\infty} A_i \cdot Mod_i \quad A_i = \langle E, Mod_i \rangle$$

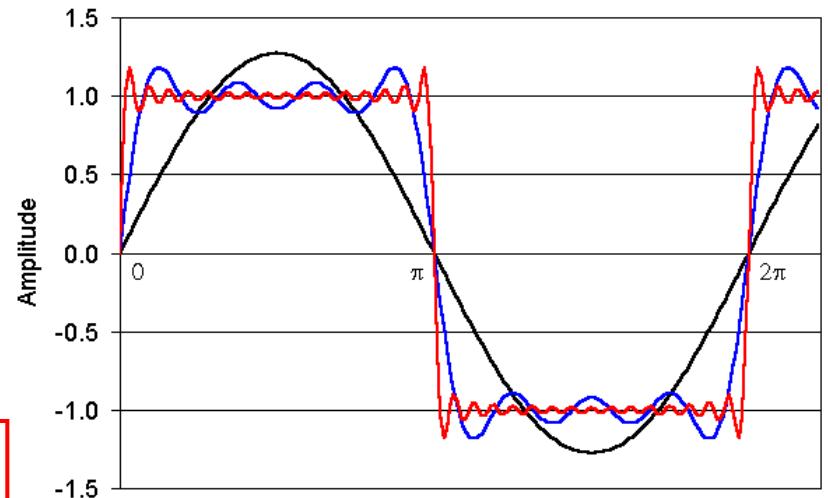
Mathematical modeling

- particular cases where analytical solution exists
 - harmonic signals, Fourier Transform, frequency spectrum

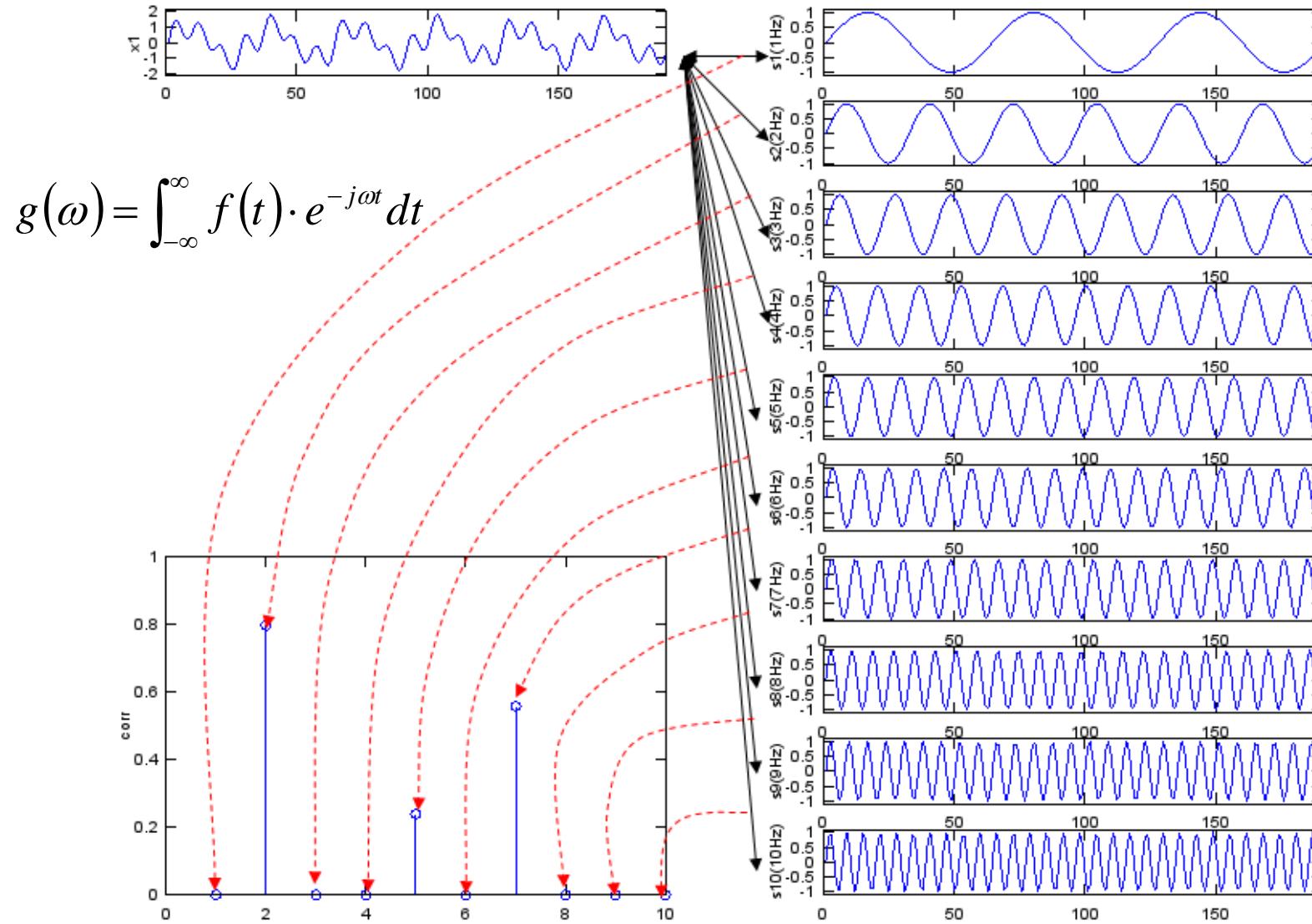
$$X = X_0 e^{j \cdot \omega \cdot t} \quad \frac{\partial X}{\partial t} = j \cdot \omega \cdot X \quad g(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt \quad f(t) = \int_{-\infty}^{\infty} g(\omega) \cdot e^{j\omega t} d\omega$$



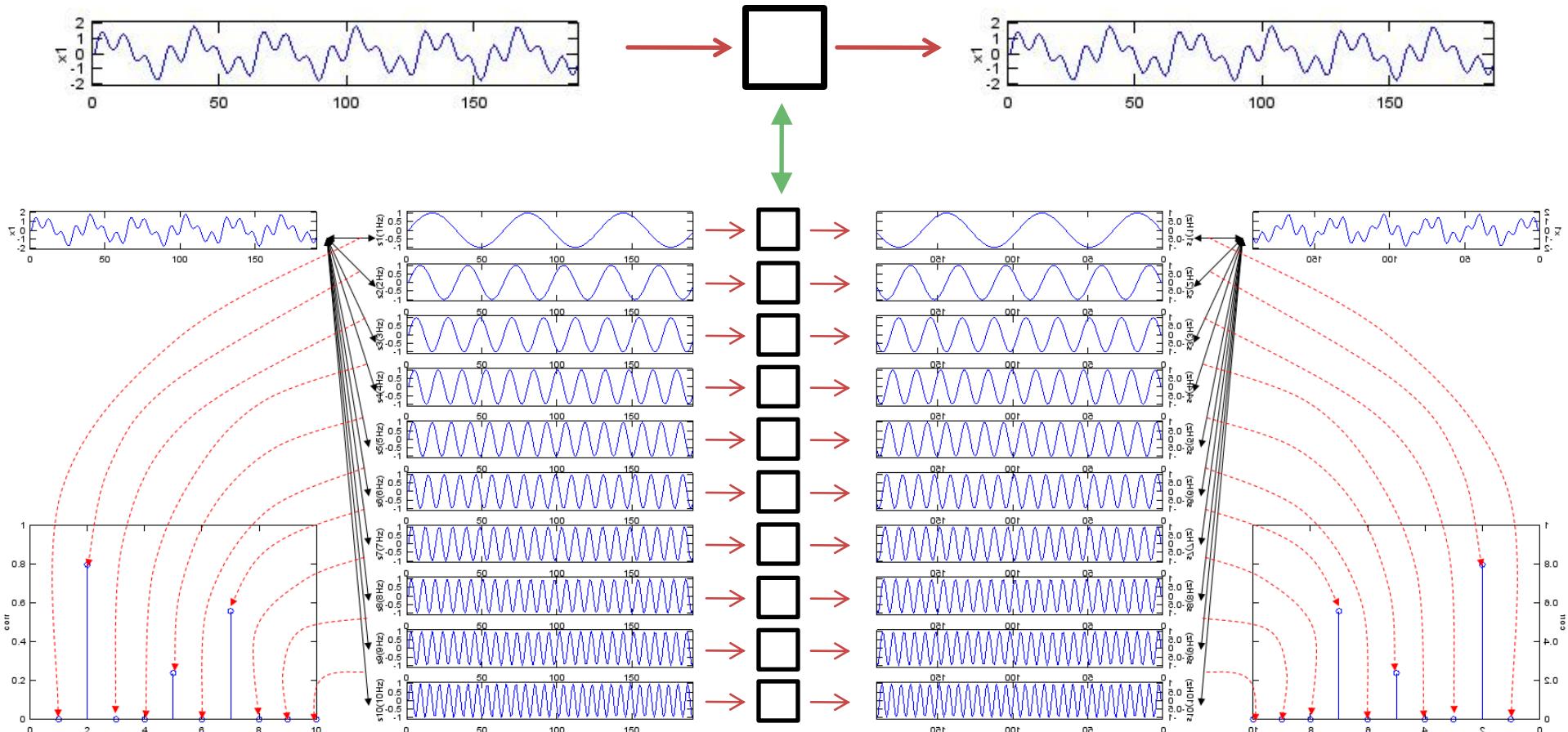
$$G(\omega)[F(\omega)]$$



Mathematical modeling



Mathematical modeling



$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt$$

$$G(\omega)[F(\omega)]$$

$$g(t) = \int_{-\infty}^{\infty} G(\omega) \cdot e^{j\omega t} d\omega$$

Mathematical modeling

- particular cases where analytical solution exists

- wave in a single direction $E^+(E^+)$, $E^-(E^-)$

- wave

- incident

$$E_y = E^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t - \beta \cdot z)} + E^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t + \beta \cdot z)}$$

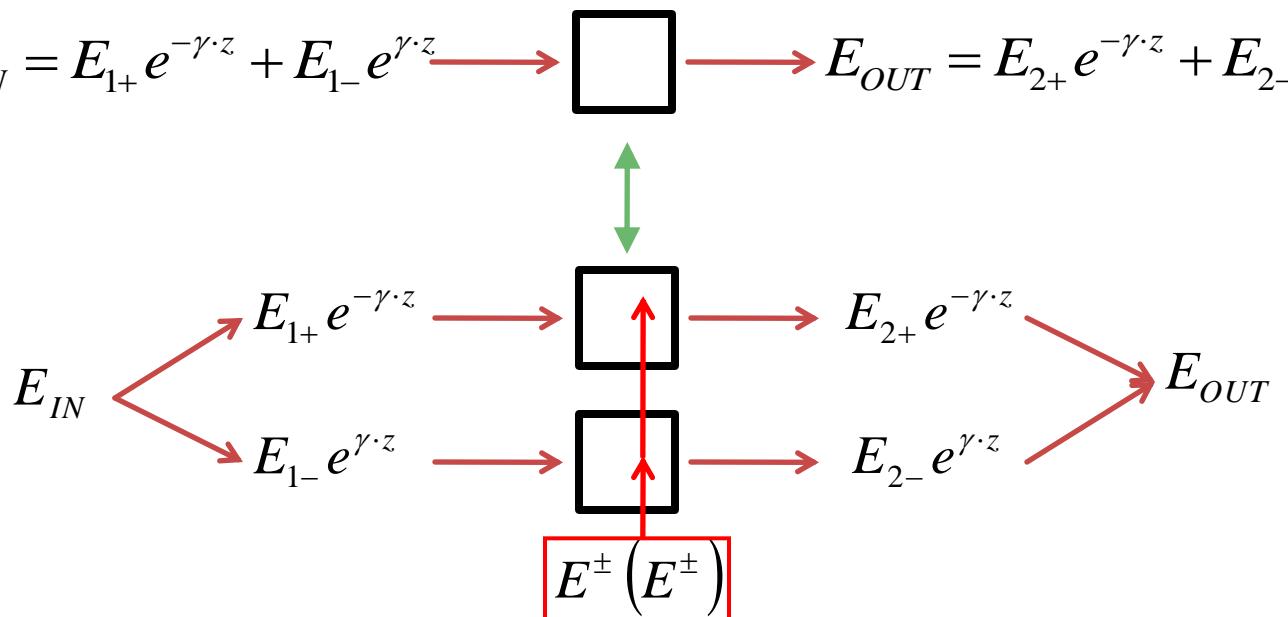
- reflected

$$E_{IN} = E_{1+} e^{-\gamma \cdot z} + E_{1-} e^{\gamma \cdot z} \rightarrow \boxed{\text{ }} \rightarrow E_{OUT} = E_{2+} e^{-\gamma \cdot z} + E_{2-} e^{\gamma \cdot z}$$

- wave

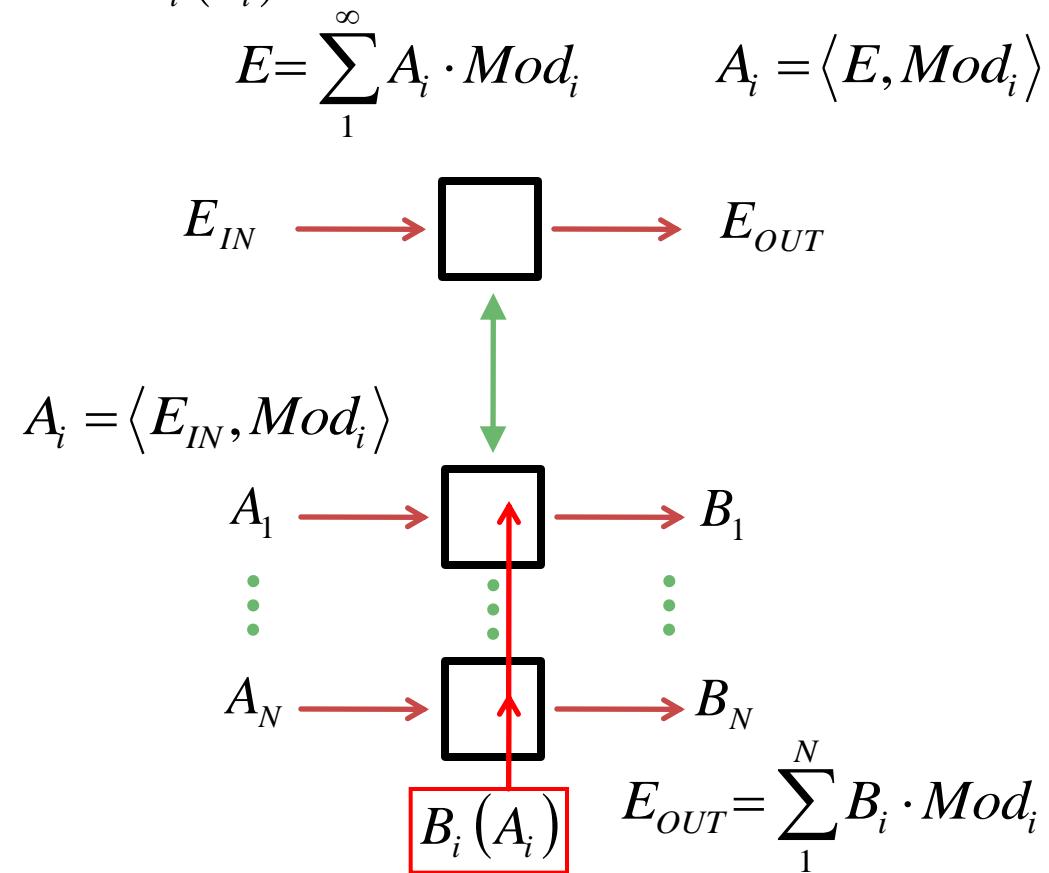
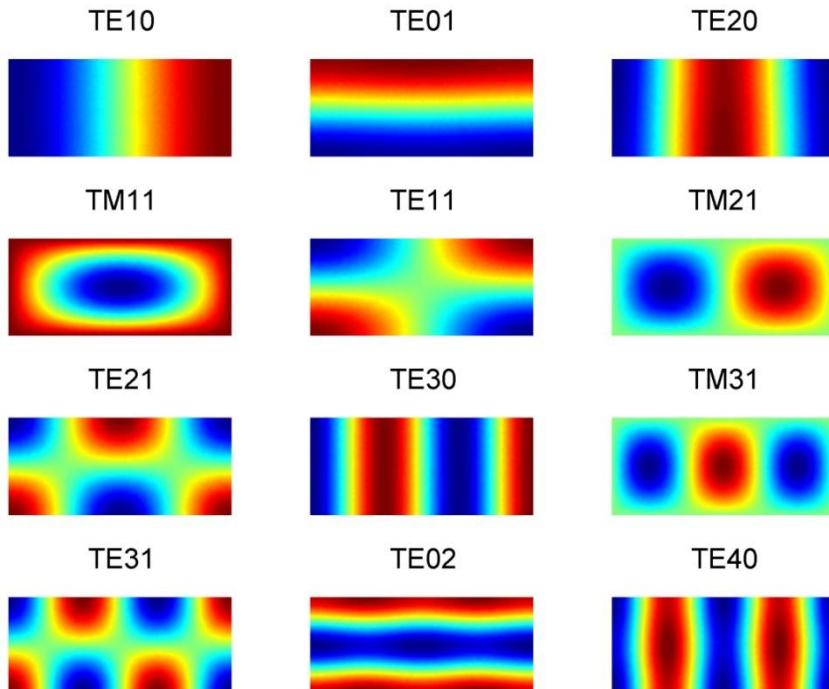
- direct

- inverse



Mathematical modeling

- particular cases where analytical solution exists
 - modes in delimited media $B_i(A_i)$



TEM transmission lines

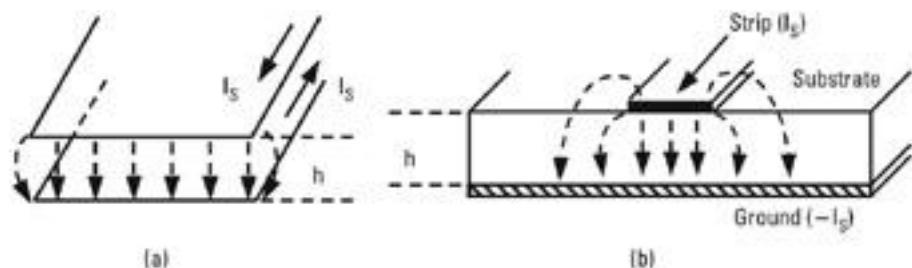
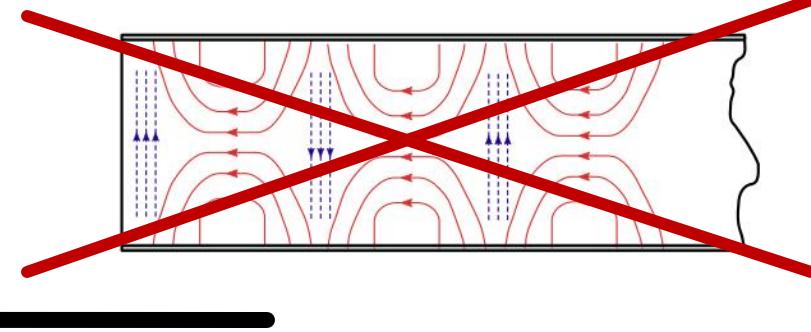
Transmission line

- TEM wave propagation, at least two conductors

$$I(z, t)$$



$$V(z, t)$$

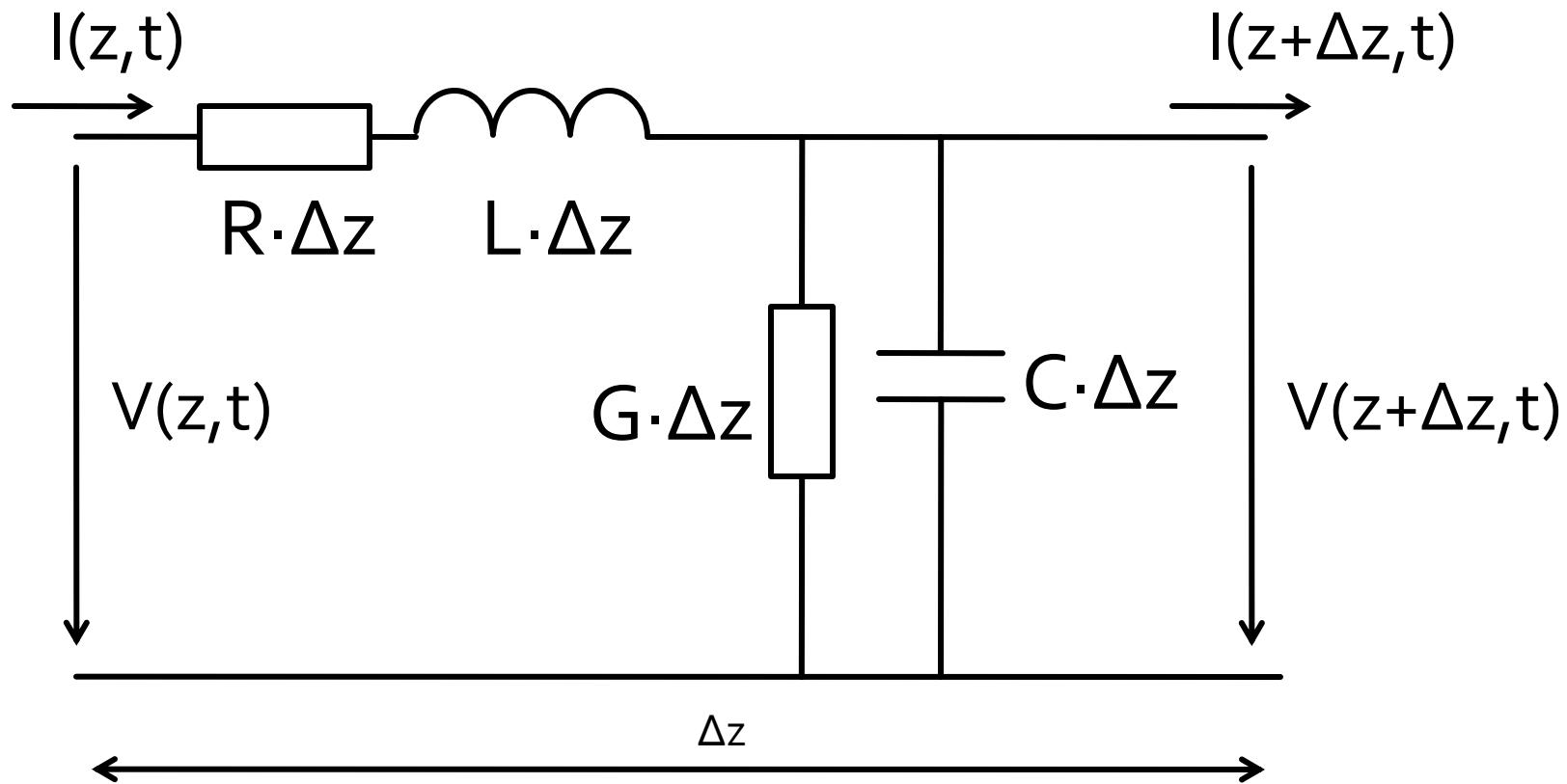


(a)

(b)

Transmission line equivalent model

- TEM wave propagation, at least two conductors



Telegrapher's equations

- time domain

$$\frac{\partial v(z,t)}{\partial z} = -R \cdot i(z,t) - L \cdot \frac{\partial i(z,t)}{\partial t} \quad K \parallel$$

$$\frac{\partial i(z,t)}{\partial z} = -G \cdot v(z,t) - C \cdot \frac{\partial v(z,t)}{\partial t} \quad K \perp$$

- harmonic signals (frequency domain)

$$\frac{dV(z)}{dz} = -(R + j \cdot \omega \cdot L) \cdot I(z) \quad \left/ \frac{d}{dz} (\dots) \right.$$

$$\frac{dI(z)}{dz} = -(G + j \cdot \omega \cdot C) \cdot V(z)$$

Solving T's E

$$\frac{d^2V(z)}{dz^2} - \gamma^2 \cdot V(z) = 0$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 \cdot I(z) = 0$$



$$\nabla^2 E - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0$$

$$\gamma^2 = -\omega^2 \epsilon \mu + j \omega \mu \sigma$$

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)}$$

$$E_y = E_+ e^{-\gamma \cdot z} + E_- e^{\gamma \cdot z}$$

Solutions

$$\left\{ \begin{array}{l} V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \\ I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \end{array} \right.$$

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$\frac{dV(z)}{dz} = -(R + j \cdot \omega \cdot L) \cdot I(z)$$

$$Z_0 \equiv \frac{R + j \cdot \omega \cdot L}{\gamma} = \sqrt{\frac{R + j \cdot \omega \cdot L}{G + j \cdot \omega \cdot C}}$$

$$\frac{V_0^+}{I_0^+} = Z_0 = -\frac{V_0^-}{I_0^-}$$

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)}$$

$$I(z) = \frac{\gamma}{R + j \cdot \omega \cdot L} (V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z})$$

- Characteristic impedance of the line

$$\lambda = \frac{2\pi}{\beta} \quad v_f = \frac{\omega}{\beta} = \lambda \cdot f$$

The lossless line

- **Lossless:** $R=G=0$

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)} = j \cdot \omega \cdot \sqrt{L \cdot C}$$

$$\alpha = 0 \quad ; \quad \beta = \omega \cdot \sqrt{L \cdot C}$$

$$Z_0 = \sqrt{\frac{R + j \cdot \omega \cdot L}{G + j \cdot \omega \cdot C}} = \sqrt{\frac{L}{C}}$$

- Z_0 is **real**

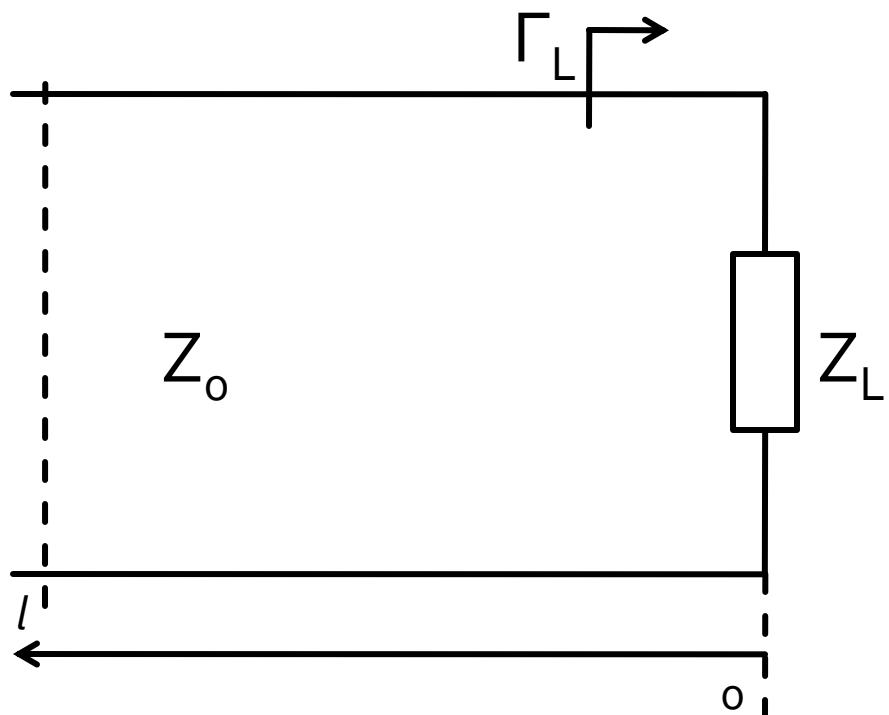
$$V(z) = V_0^+ e^{-j \cdot \beta \cdot z} + V_0^- e^{j \cdot \beta \cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j \cdot \beta \cdot z} - \frac{V_0^-}{Z_0} e^{j \cdot \beta \cdot z}$$

$$\lambda = \frac{2\pi}{\omega \cdot \sqrt{LC}}$$

$$v_f = \frac{1}{\sqrt{LC}}$$

The lossless line



$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$$

$$Z_L = \frac{V(0)}{I(0)} \quad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

- voltage reflection coefficient

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

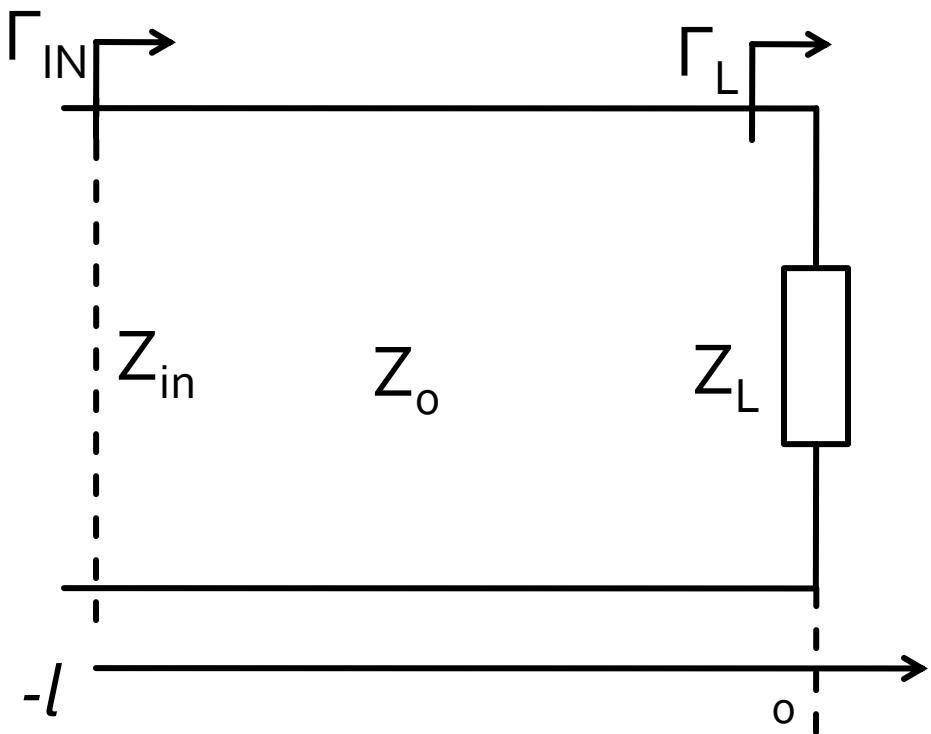
- Z_0 real

The lossless line

- voltage reflection coefficient seen at the input of the line

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$\Gamma = \Gamma(z) = \frac{V_0^-(z)}{V_0^+(z)}$$



$$V(0) = V_0^+ + V_0^-$$

$$\Gamma(0) = \Gamma_L = \frac{V_0^-}{V_0^+}$$

$$V(-l) = V_0^+ e^{j\beta l} + V_0^- e^{-j\beta l}$$

$$\Gamma(-l) = \Gamma_{IN} = \frac{V_0^- \cdot e^{-j\beta l}}{V_0^+ \cdot e^{j\beta l}} = \Gamma(0) \cdot e^{-2j\beta l}$$

$$|\Gamma(-l)| = |\Gamma(0)| \cdot |e^{-2j\beta l}| = |\Gamma(0)|$$

$$\boxed{\Gamma_{IN} = \Gamma_L \cdot e^{-2j\beta l}}$$

$$\boxed{|\Gamma_{IN}| = |\Gamma_L|}$$

The lossless line

$$V(z) = V_0^+ \cdot (e^{-j\beta z} + \Gamma \cdot e^{j\beta z}) \quad I(z) = \frac{V_0^+}{Z_0} \cdot (e^{-j\beta z} - \Gamma \cdot e^{j\beta z})$$

- time-average Power flow along the line

$$P_{avg} = \frac{1}{2} \cdot \text{Re}\{V(z) \cdot I(z)^*\} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot \text{Re}\left\{1 - \Gamma^* \cdot e^{-2j\beta z} + \Gamma \cdot e^{2j\beta z} - |\Gamma|^2\right\}$$
$$P_{avg} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot \left(1 - |\Gamma|^2\right)$$

$(z - z^*) = \text{Im}$

- Total power delivered to the load = Incident power – “Reflected” power
- Return “Loss” [dB]
$$\text{RL} = -20 \cdot \log|\Gamma| \quad [\text{dB}]$$

Polar representation

- Euler's formula

$$e^{j \cdot x} = \cos x + j \cdot \sin x; \forall x \in R$$

- Polar representation

$$z = a + j \cdot b = |z| \cdot e^{j \cdot \varphi}$$

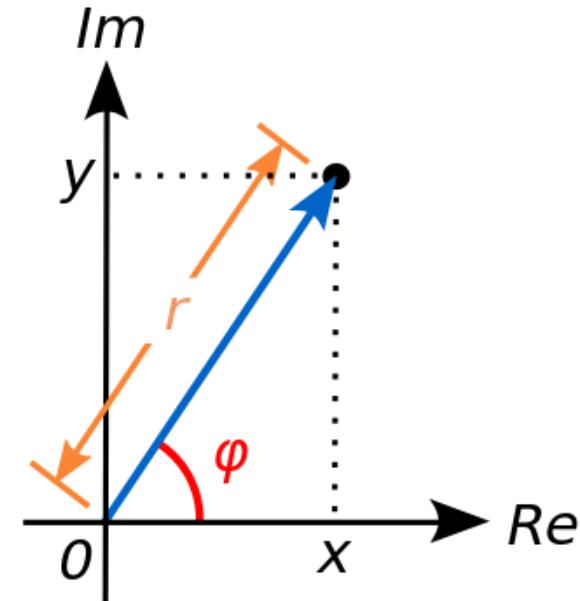
$$z = a + j \cdot b = |z| \cdot (\cos \varphi + j \cdot \sin \varphi)$$

$$z^n = (|z| \cdot e^{j \cdot \varphi})^n = |z|^n \cdot e^{j \cdot n \cdot \varphi} = |z|^n \cdot [\cos(n \cdot \varphi) + j \cdot \sin(n \cdot \varphi)]$$

→ $\sqrt{z} = (|z| \cdot e^{j \cdot \varphi})^{1/2} = \sqrt{|z|} \cdot e^{j \cdot \frac{\varphi}{2}} = \sqrt{|z|} \cdot \left(\cos \frac{\varphi}{2} + j \cdot \sin \frac{\varphi}{2} \right)$

$$z \cdot w = |z| \cdot e^{j \cdot \varphi} \cdot |w| \cdot e^{j \cdot \theta} = |z| \cdot |w| \cdot e^{j \cdot (\varphi + \theta)} = |z| \cdot |w| \cdot [\cos(\varphi + \theta) + j \cdot \sin(\varphi + \theta)]$$

$$z/w = \frac{|z| \cdot e^{j \cdot \varphi}}{|w| \cdot e^{j \cdot \theta}} = \frac{|z|}{|w|} \cdot e^{j \cdot \varphi} \cdot e^{-j \cdot \theta} = \frac{|z|}{|w|} \cdot [\cos(\varphi - \theta) + j \cdot \sin(\varphi - \theta)]$$



Polar representation

■ Euler's formula

$$e^{j \cdot x} = \cos x + j \cdot \sin x; \forall x \in R$$

$$e^{j \cdot x} + e^{-j \cdot x} = \cos x + j \cdot \sin x + \cos(-x) + j \cdot \sin(-x)$$

$$e^{j \cdot x} + e^{-j \cdot x} = \cos x + j \cdot \sin x + \cos x - j \cdot \sin x = 2 \cdot \cos x$$

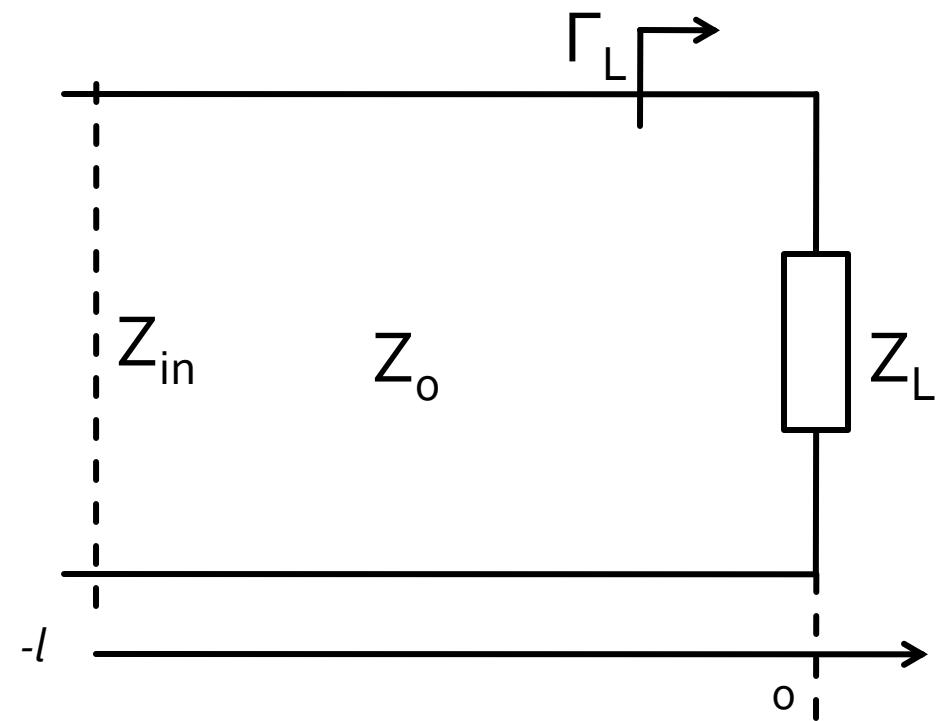

$$\cos x = \frac{e^{j \cdot x} + e^{-j \cdot x}}{2}$$

$$e^{j \cdot x} - e^{-j \cdot x} = \cos x + j \cdot \sin x - \cos(-x) - j \cdot \sin(-x)$$

$$e^{j \cdot x} - e^{-j \cdot x} = \cos x + j \cdot \sin x - \cos x + j \cdot \sin x = 2j \cdot \sin x$$


$$\sin x = \frac{e^{j \cdot x} - e^{-j \cdot x}}{2j}$$

The lossless line



$$V(-l) = V_0^+ e^{j \cdot \beta \cdot l} + V_0^- e^{-j \cdot \beta \cdot l}$$

$$I(-l) = \frac{V_0^+}{Z_0} e^{j \cdot \beta \cdot l} - \frac{V_0^-}{Z_0} e^{-j \cdot \beta \cdot l}$$

$$Z_{in} = \frac{V(-l)}{I(-l)} \quad Z_{in} = Z_0 \cdot \frac{1 + \Gamma \cdot e^{-2j \cdot \beta \cdot l}}{1 - \Gamma \cdot e^{-2j \cdot \beta \cdot l}}$$

- the **input impedance** seen looking toward the load

$$Z_{in} = Z_0 \cdot \frac{(Z_L + Z_0) \cdot e^{j \cdot \beta \cdot l} + (Z_L - Z_0) \cdot e^{-j \cdot \beta \cdot l}}{(Z_L + Z_0) \cdot e^{j \cdot \beta \cdot l} - (Z_L - Z_0) \cdot e^{-j \cdot \beta \cdot l}}$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

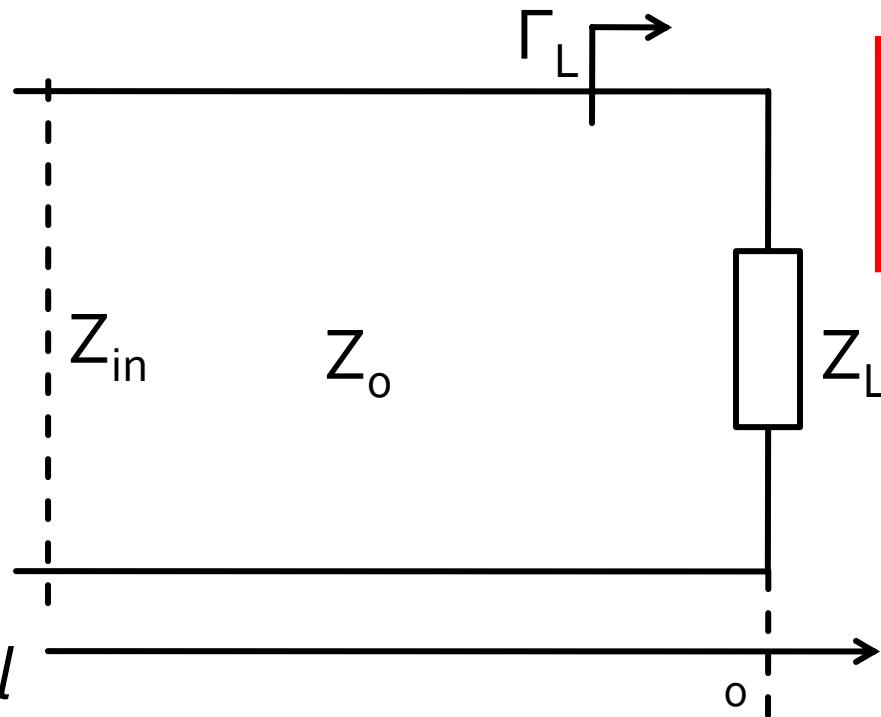
The lossless line

- the **input impedance** seen looking toward the load

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

The lossless line

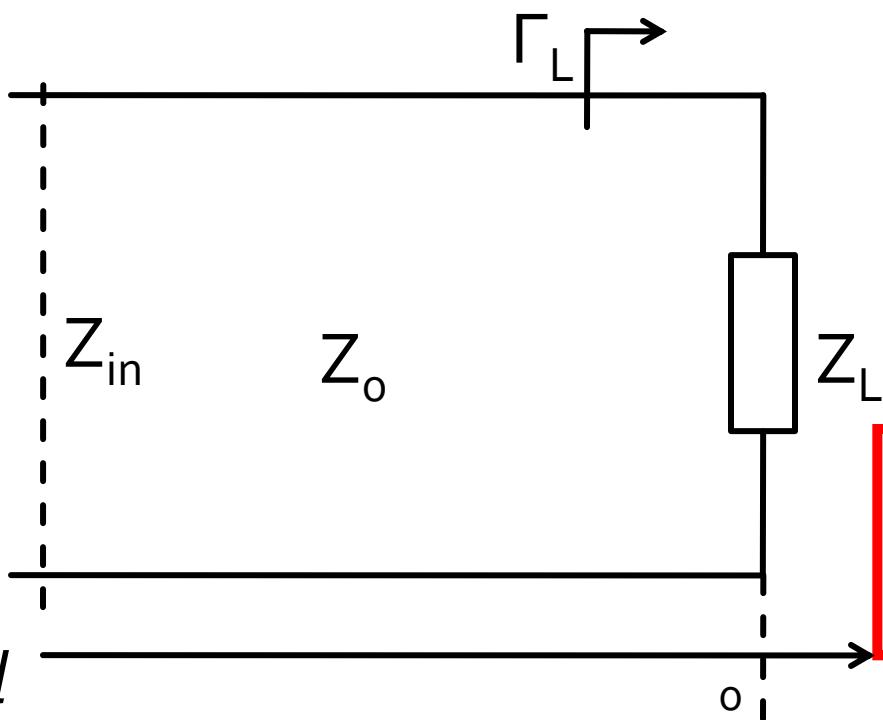
- input impedance of a length l of transmission line with characteristic impedance Z_0 , loaded with an arbitrary impedance Z_L



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

The lossless line

- input impedance is **frequency dependent** through $\beta \cdot l$



$$v_f = \frac{\omega}{\beta} = \lambda \cdot f \quad \lambda = \frac{2\pi}{\beta}$$

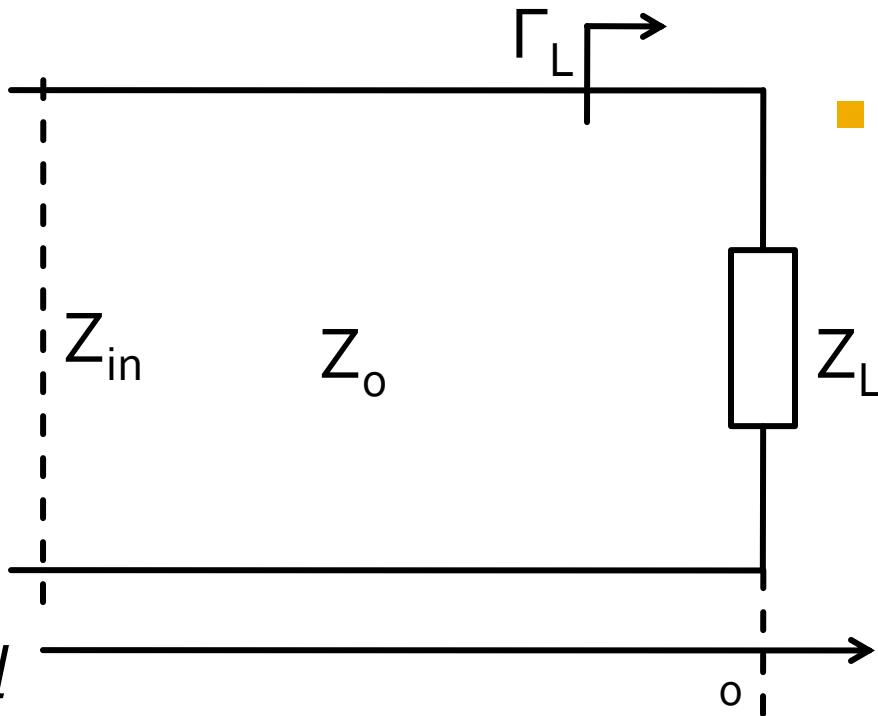
$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot l = \frac{2\pi \cdot f}{v_f} \cdot l = \frac{2\pi \cdot l}{v_f} \cdot f$$

frequency dependence is **periodical**, imposed by the tan trigonometric function

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

The lossless line, special cases

- $l = k \cdot \lambda / 2$ $\beta \cdot l = \frac{2\pi}{\lambda} \cdot l = k \cdot \pi$ $\tan \beta \cdot l = 0$ $Z_{in} = Z_0$
- $l = \lambda / 4 + k \cdot \lambda / 2$ $\tan \beta \cdot l \rightarrow \infty$ $Z_{in} = \frac{Z_0^2}{Z_L}$



■ quarter-wave transformer

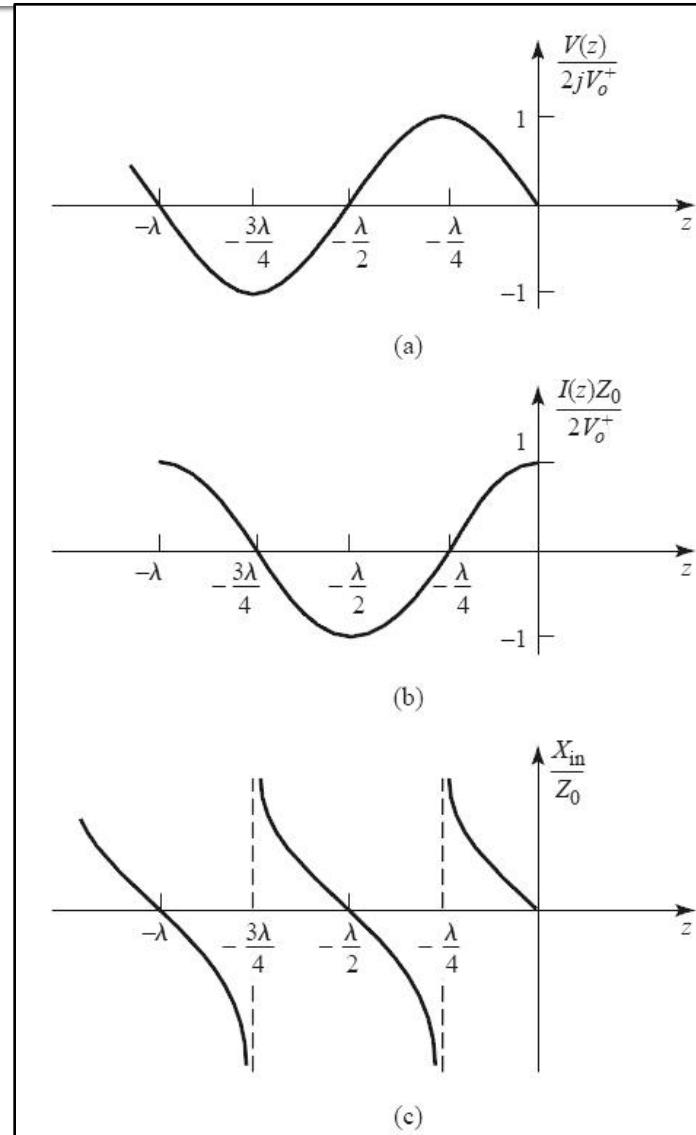
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

Short-circuited transmission line

- $Z_L = 0$
- purely imaginary for any length l
 - $\pm \rightarrow$ depending on l value

$$Z_{in} = j \cdot Z_0 \cdot \tan \beta \cdot l$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

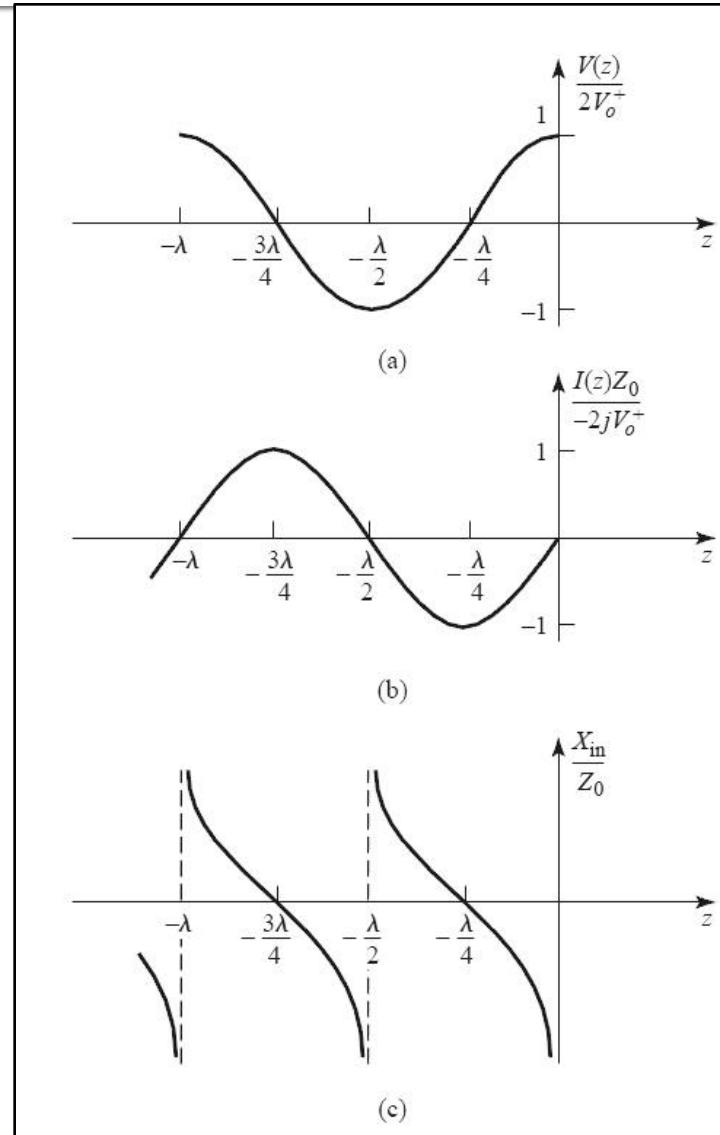


Open-circuited transmission line

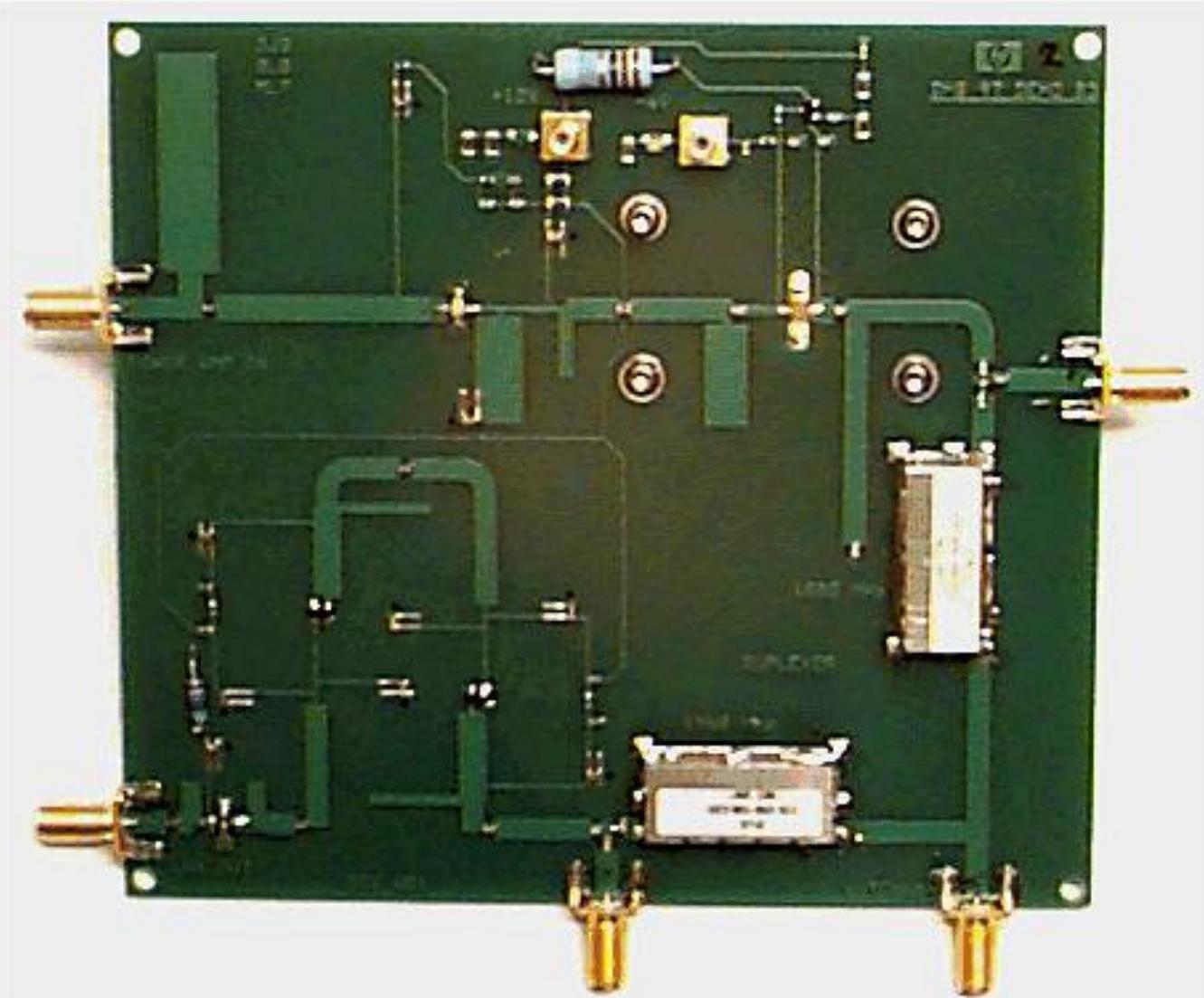
- $Z_L = \infty \rightarrow 1/Z_L = 0$
- purely imaginary for any length l
 - $\pm/- \rightarrow$ depending on l value

$$Z_{in} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

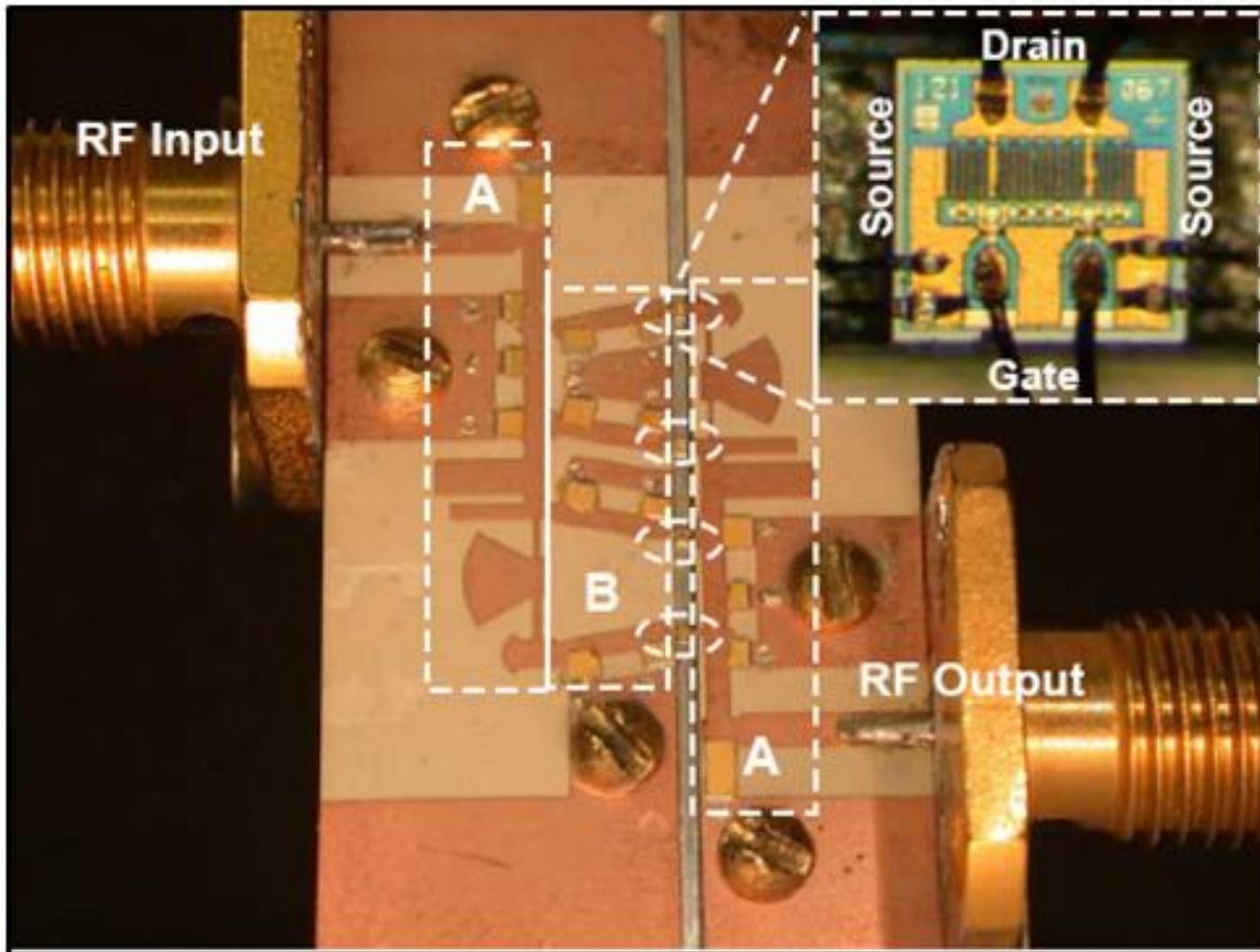
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$



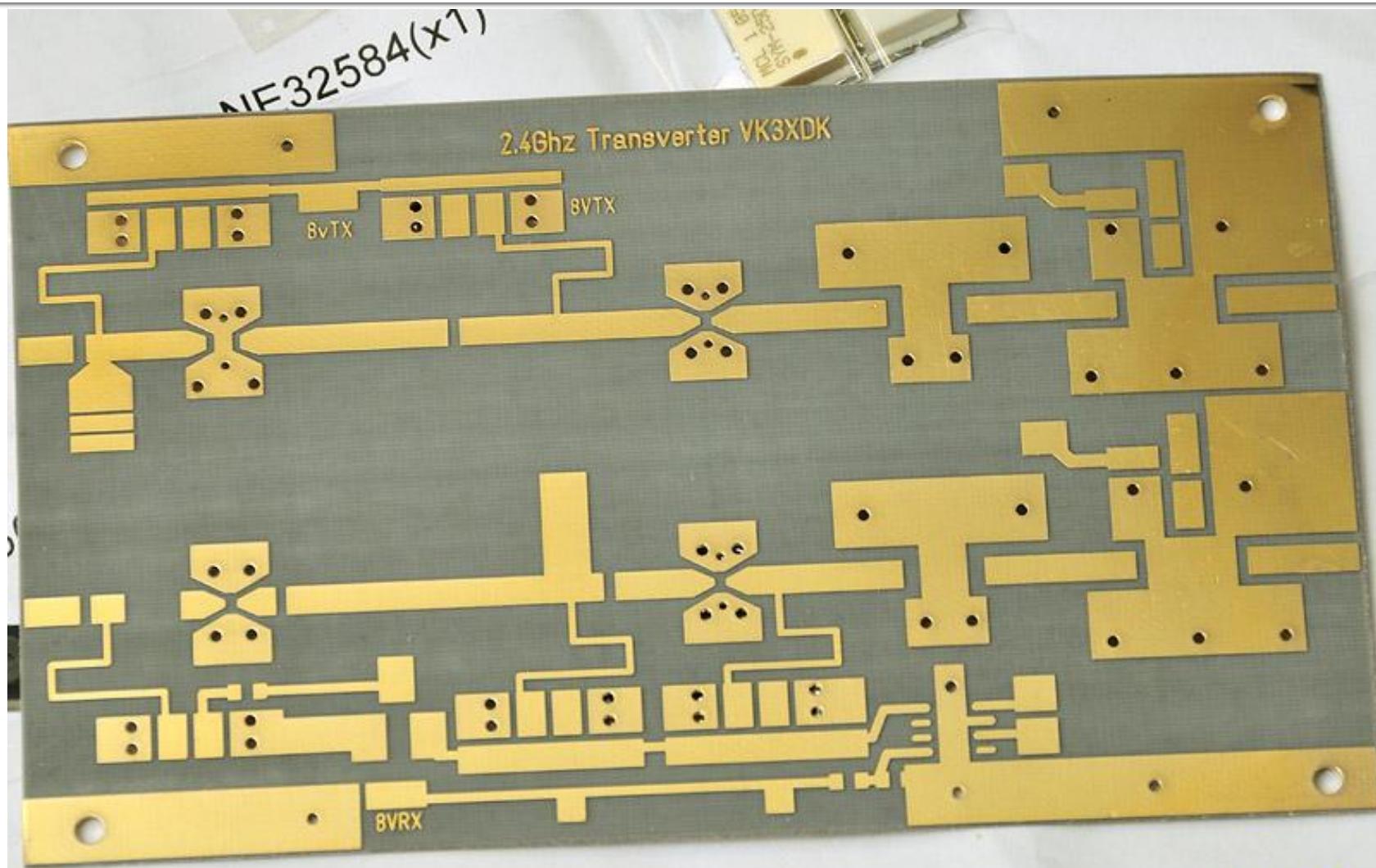
Examples



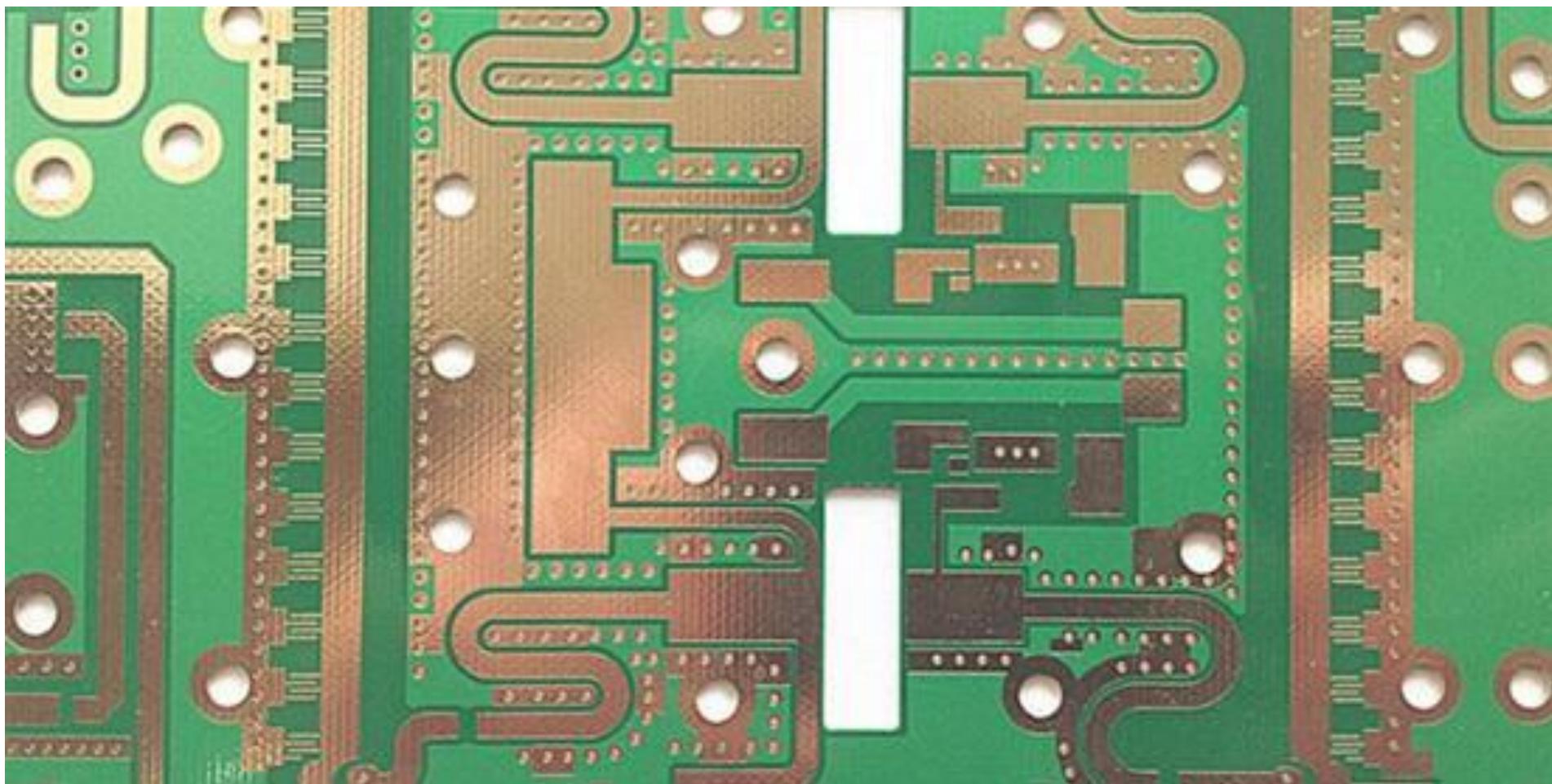
Examples



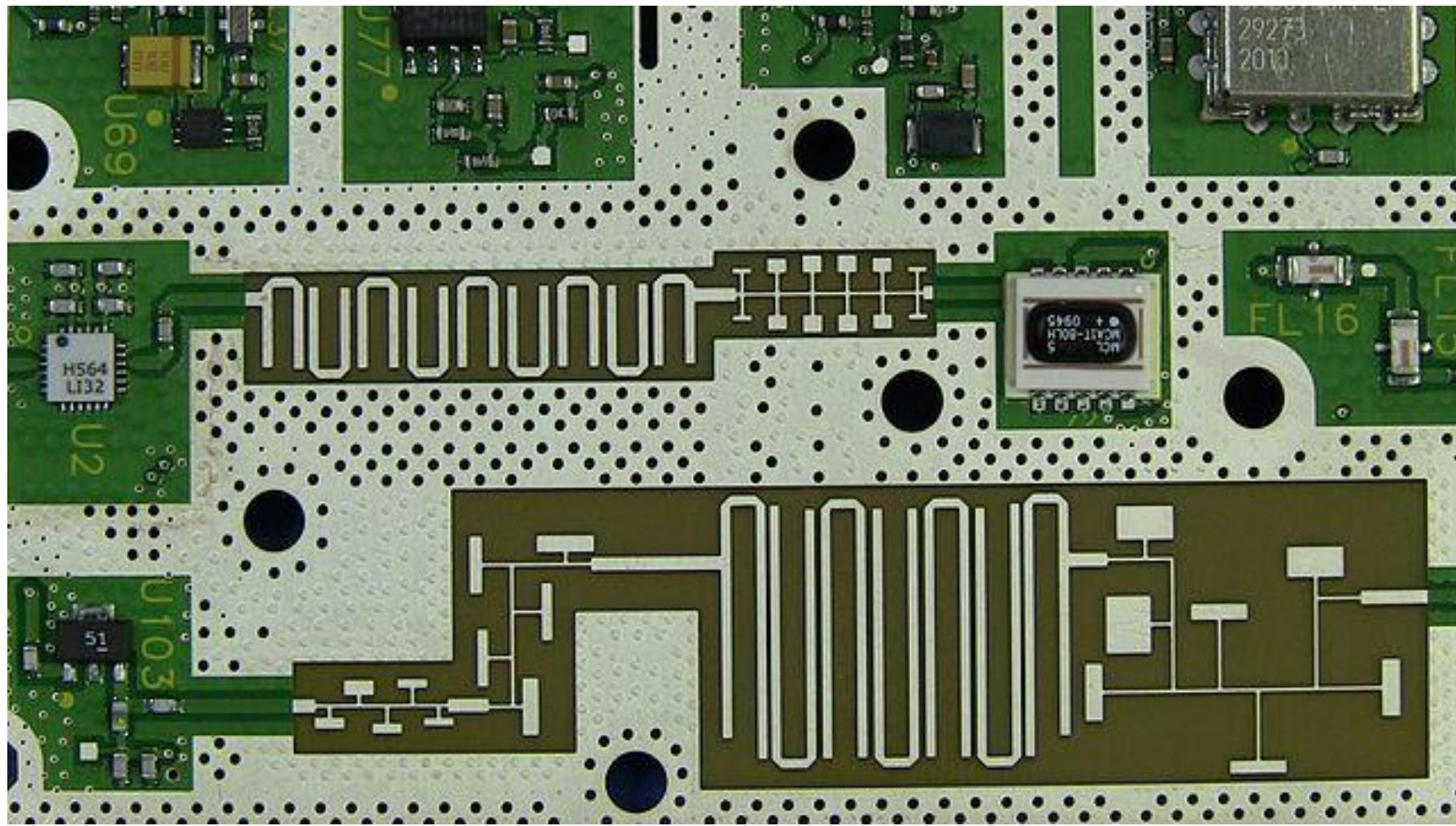
Examples



Examples



Examples



Voltage standing wave ratio

$$V(z) = V_0^+ \cdot (e^{-j\beta z} + \Gamma \cdot e^{j\beta z}) \quad |V(z)| = |V_0^+| \cdot |e^{-j\beta z}| \cdot |1 + \Gamma \cdot e^{2j\beta z}| \quad \Gamma = |\Gamma| \cdot e^{j\theta}$$

$$|V(z)| = |V_0^+| \cdot |1 + |\Gamma| \cdot e^{\theta + 2j\beta z}|$$

maximum magnitude value for $e^{\theta + 2j\beta z} = 1$

$$V_{\max} = |V_0^+| \cdot (1 + |\Gamma|)$$

minimum magnitude value for $e^{\theta + 2j\beta z} = -1$

$$V_{\min} = |V_0^+| \cdot (1 - |\Gamma|)$$

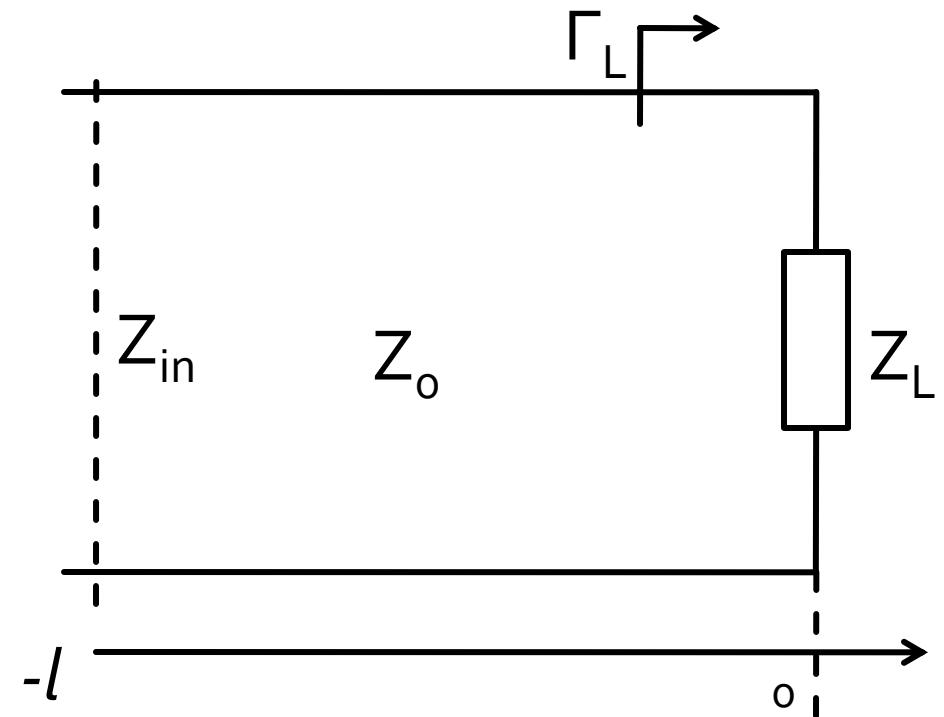
- SWR is defined as the ratio between maximum and minimum

- (Voltage) Standing Wave Ratio

$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

- real number $1 \leq VSWR < \infty$
 - a measure of the mismatch (SWR = 1 means a matched line)

The lossless line +/-



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

$$V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z}$$

$$I(z) = I_0^+ e^{-\gamma \cdot z} + I_0^- e^{\gamma \cdot z}$$

$$\Gamma(-l) = \Gamma(0) \cdot e^{-2j\beta l}$$

$$\Gamma_{in} = \Gamma_L \cdot e^{-2j\beta l}$$

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